P. Vojta

Math 54M First Midterm

80 minutes

Spring 2001

[This was a fairly easy exam: median was 45 out of a possible 50.]

1. (7 points) Compute (or, if it is not defined, say so):

   (a). $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$

   (b). $A^{10}B$, where $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

   (c). $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 7 & 7 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

   (d). $3\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - 4\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -10 & 4 \\ 2 & 1 \end{bmatrix}$

   (e). The distance between $(9, 4, 7, 2, 0)$ and $(6, 2, 8, 0, 1)$ in $\mathbb{R}^5$.

   (f). $\mathbf{u} \cdot \mathbf{v}$, where $\mathbf{u} = (9, 4, 7, 2, 0)$ and $\mathbf{v} = (1, -2, 3, 2, 8)$

2. (7 points) Solve the system:

$$\begin{aligned} x + 3y + z + u - v &= 0 \\ 2y + 3z - 4u + 3v &= 3 \\ 2u + v &= 3 \\ v &= 2 \end{aligned}$$

3. (10 points) Invert the matrix $\begin{bmatrix} 0 & 1 & -2 \\ 1 & 3 & -4 \\ -3 & -7 & 10 \end{bmatrix}$.

4. (6 points) State the definition of an invertible matrix, and give three conditions that are equivalent to invertibility.

5. (5 points) Find the projection of $(1, 1, 0)$ on $(2, 1, -2)$.

6. (5 points) Is $\mathbb{Z}$ (= the set of all integers) a subspace of $\mathbb{R} = \mathbb{R}^1$? Justify your answer.

7. (10 points) Are the vectors

$$(1, 0, 0), \quad (2, 4, 7) \quad (7, 3, 5)$$

   linearly independent? Justify your answer.