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Spring 1996, Math 54, Lecture 2
First Midterm

13 February, 1996
9:40-11:00 AM

1. (3 points each = 24 points) Compute each of the following if it is defined. If it is undefined, say so.

(i) A^2, where $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ (ii) $p(B)$, where $p(x) = x^2 - x - 1$ and $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$

(iii) $\|\mathbf{x}\|$, where $\mathbf{x} = \begin{bmatrix} 2 \\ -3 \\ 6 \end{bmatrix}$ (iv) AA^T, where $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 3 \end{bmatrix}$

(v) $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} - \begin{bmatrix} 1 \\ 9 \\ 9 \\ 6 \end{bmatrix}$ (vi) $\det \begin{bmatrix} 2 & 4 & -3 \\ 0 & 5 & 7 \\ 0 & 0 & 10 \end{bmatrix}$ (vii) $\begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

(viii) $\operatorname{tr} \begin{bmatrix} 1 & 2 & 0 \\ 3 & 4 & 0 \\ 5 & 6 & 0 \end{bmatrix}$

2. (18 points) Suppose A and B are $n \times n$ matrices, and $\mathbf{x} \in R^n$ a vector which is both an eigenvector of A, corresponding to the eigenvalue λ_1, and an eigenvector of B, corresponding to the eigenvalue λ_2. Prove that $\mathbf{x}$ is also an eigenvector of AB, corresponding to the eigenvalue $\lambda_1 \lambda_2$.

3. (25 points) Consider the system of linear equations

$$\begin{aligned} 2x_1 + 3x_2 + 4x_3 &= 6 \\ 3x_1 + 4x_2 + 5x_3 &= 8 \\ 4x_1 + 5x_2 + 6x_3 &= 10 \end{aligned}$$

(a) (15 points) Give the augmented matrix for the above system of equations, and find the reduced row-echelon form of that matrix

(b) (10 points) Find all solutions to the above system of equations. (This is most easily done using the result of (a), but the correct answer gotten by any method will be accepted.)

4. (6 points each = 18 points) Define (using words, formulas, or both) the following concepts.

(a) The inverse of an $n \times n$ matrix A (assuming A has an inverse).

(b) The linear transformation T_A corresponding to an $m \times n$ matrix A. (Your answer should make clear, implicitly or explicitly, the domain and codomain of this transformation.)

(c) The standard basis of R^4.

5. (15 points) Given that the matrix $\begin{bmatrix} -1 & 1 \\ 8 & 1 \end{bmatrix}$ has 3 as an eigenvalue, find an eigenvector of this matrix corresponding to that eigenvalue.