

22 Warren and 4 Leconte
1:10–2 PM

Your Name: _____ TA: _____

Please check that you have all 5 pages of this exam booklet. Write your name on *each* page. As you turn through the pages, look for the easy questions—do them first. Remember that this exam is only 50 minutes long.

- You need not simplify your answers unless you are specifically asked to do so.
- It is essential to write legibly and *show your work*.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.
- Be especially sure to provide some explanation when you do the “true or false” problems.

During this exam, you are not allowed to use calculators or consult your notes or books.

Problem	Maximum	Your Score
1	10	
2	13	
3	10	
4	12	
Total	45	

At the conclusion of the exam, hand in this exam paper to your TA.

Your Name: _____

1a (5 points). True or false: If a function f maps a set to itself, then f is one-to-one if and only if f is onto.

1b (5 points). True or false: If p and q are propositions, then $\neg p \rightarrow q$ and $p \wedge q$ are equivalent.

Your Name: _____

2a (*5 points*). In the sleepy town of Cognac, 33 cars have license plates containing **TI**, while 109 have license plates containing the string **NY**. Suppose that 115 cars have license plates containing either **TI** or **NY**. How many cars have plates with **TI** or **NY**, but not both?

2b (*8 points*). Use the Euclidean algorithm to solve the congruence $5x \equiv 1 \pmod{13}$.

Your Name: _____

3 (*10 points*). The celebrated Ribonacci numbers R_n are defined as follows:

$$R_0 = 0; \quad R_1 = 1; \quad R_n = 3R_{n-1} - 2R_{n-2} \text{ for } n \geq 2.$$

Prove that $R_n = 2^n - 1$ for all $n \geq 0$.

Your Name: _____

4a (5 points). True or false: If $f(x) = \log_{\pi} x$ and $g(x) = \log_{10} x$, then $f(x) = O(g(x))$.

4b (7 points). Find a non-negative integer x satisfying the two congruences

$$\begin{cases} x \equiv 15 \pmod{18} \\ x \equiv 47 \pmod{50}. \end{cases}$$

[Hint: first find a solution with x negative.]