UNIVERSITY OF CALIFORNIA
College of Engineering
Departments of Mechanical Engineering and Material Science & Engineering

Fall 2006 Prof. Ritchie

MSEc113/MEc124
Mechanical Behavior of Materials

Midterm #1 September 19th 2006 Solutions - 40 points

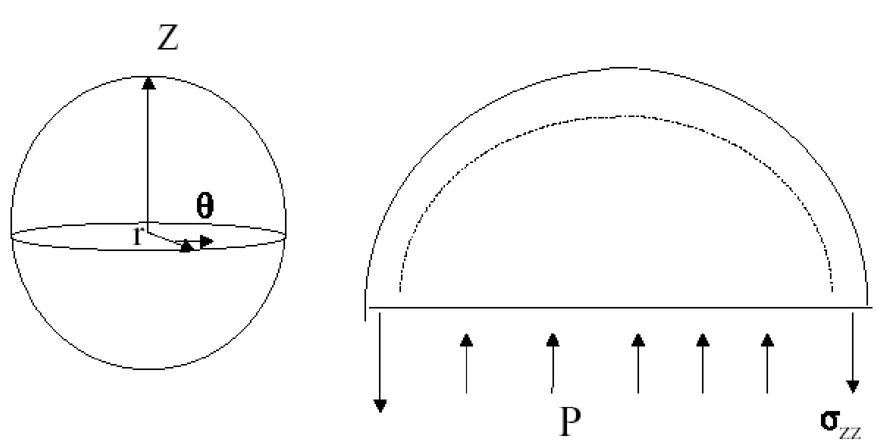
Problem 1

At your new job in at a Silicon Valley manufacturer of semiconductors, your first task is to perform a safety evaluation of a pressure vessel. The vessel is spherical in shape with a diameter of 2m and will be used to store highly toxic pyrophoric silane (SiH_4) gas (using in the deposition of silicon thin films). The vessel is stored outside the processing laboratory for safety reasons. If the container fails, most of Santa Clara County will perish. In your evaluation you must answer the following questions:

a) Use the Tresca Criterion to determine what internal pressure will cause first yielding in the 5mm thick walls if the vessel is made from a carbon steel (uniaxial tensile properties: E = 210 GPa, s_y = 450 MPa, s_u = 560 MPa)?

b) What are the principal stresses and the maximum shear stress at the maximum operating pressure of 1800 kPa?

Solution 1 - 10 points



The stresses in the spherical vessel can be found by establishing equilibrium for a section of the sphere as shown above:
$P\pi r^2 = \sigma_{zz} 2\pi rt$, $\sigma_{zz} = Pr/2t$
by symmetry $\sigma_{\theta\theta} = \sigma_{zz} = Pr/2t$
For thin walled pressure vessels $\sigma_{rr} = 0$

5

a) What internal pressure will cause yielding in the 5mm thick walls if the vessel is made from a carbon steel(uniaxial tensile properties: E=210 GPa, σ_y=450 MPa,σ_u=560 MPa)?

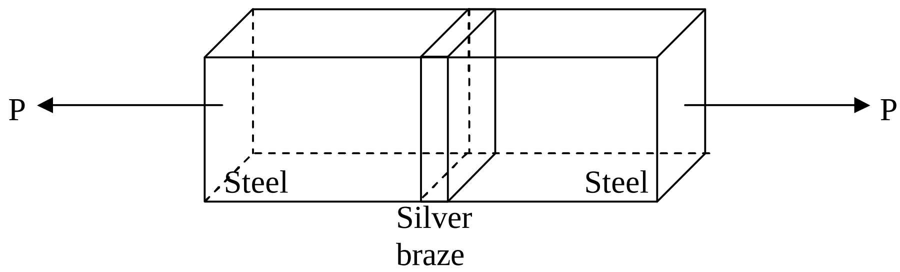
Using the Tresca Criterion:
$\tau_{max} = \tau_y = 450/2 = 225MPa$
i.e.yielding condition
Also $\tau_{max} = \sigma_{\theta\theta}/2 = Pr/4t$
$P_{yield} = 4t\tau_{max}/r = 4.5MPa$

5

b)What are the principal stresses and the maximum shear stress at the maximum operating pressure of 1800 kPa?
$\sigma_{\theta\theta} = \sigma_{zz} = Pr/2t = 180MPa$
$\sigma_{rr} = 0$
$\tau_{max} = \sigma_{\theta\theta}/2 = 90MPa$

Problem 2

Two stainless steel rods with a square cross-section, 1.10 m on both side and 25 m long, are joined by a silver alloy braze (0.5 mm thick):



	E (GPa)	?	s_y (MPa)	s_{UTS} (MPa)
Stainless Steel	200	0.3	1240	1530
Silver Alloy	30.3	0.367	140	140

This structure is loaded in uniaxial tension, parallel to the long axis of the steel rods and perpendicular to the braze joint. The alignment is such that bending is not allowed to occur.

Two modes of mechanical failure modes are possible:

1) Yielding of the silver braze
2) Yielding of the steel

What is the value of the applied uniaxial force (P) required to initiate first yield in this joined-steel configuration and where will the yielding first occur?

Assumptions:

1) Because the silver braze is relatively "thin" compared to the steel bar, the deformation of the silver is controlled by the steel because it is constrained (the stiffness of the steel far exceeds that of the silver) – therefore, you can assume that the strain in the joint is the strain in the steel.
2) Hint 1: First compute the Poisson's contraction strains in the silver braze joint using Hooke's Law; then consider the corresponding strain in the steel where no constraint will exist.
3) Hint 2: Use the Von Mises criterion to calculate the potential yielding in the Silver alloy

Solution 2 - 15 points

Load to Yield Steel

$\sigma_{11} := 1240\,10^{6}$

$A := 1.1 \cdot 1.1$

$P := \sigma_{11} \cdot A$

$P = 1.5 \times 10^{9}$ Newtons

Load to Yield Silver

Triaxial Stresses develop due to constraint

$$\sigma_{11} = \frac{P}{A}$$

$$\varepsilon_{11Ag} = \frac{\sigma_{11} - \nu_{Ag} \cdot (\sigma_{22} + \sigma_{33})}{E_{Ag}}$$

$$\varepsilon_{22Ag} = \frac{\sigma_{22} - \nu_{Ag} \cdot (\sigma_{11} + \sigma_{33})}{E_{Ag}}$$

$$\varepsilon_{33Ag} = \frac{\sigma_{33} - \nu_{Ag} \cdot (\sigma_{11} + \sigma_{22})}{E_{Ag}}$$

Strain in Silver in 22 and 33 direction are controlled by the Steel.
In the 11 direction the strains are different

For Steel, since there are no constraints the only stress affecting it is th

$$\varepsilon_{22St} = \frac{-\nu_{St} \cdot \sigma_{11}}{E_{St}}$$

$$\varepsilon_{33St} = \frac{-\nu_{St} \cdot \sigma_{11}}{E_{St}}$$

$$\varepsilon_{22St} = \varepsilon_{22Ag}$$

$$\varepsilon_{33St} = \varepsilon_{33Ag}$$

Thus

$$\frac{-\nu_{St}\cdot\sigma_{11}}{E_{St}} = \frac{\sigma_{22} - \nu_{Ag}\cdot(\sigma_{11} + \sigma_{33})}{E_{Ag}}$$

$$\frac{-\nu_{St}\cdot\sigma_{11}}{E_{St}} = \frac{\sigma_{33} - \nu_{Ag}\cdot(\sigma_{11} + \sigma_{22})}{E_{Ag}}$$

$$\sigma_{22} = \sigma_{33}$$

Which then simplifies to:

$$\sigma_{22} = \sigma_{33} = \frac{\sigma_{11}\cdot\left(\nu_{Ag} - \nu_{St}\cdot\frac{E_{Ag}}{E_{St}}\right)}{1 - \nu_{Ag}}$$

To find P use the Von Mises criteria and the known yield strength

$$\sigma_{ysAg} = \frac{1}{\sqrt{2}}\cdot\sqrt{(\sigma_{11} - \sigma_{22})^2 + (\sigma_{11} - \sigma_{33})^2 + (\sigma_{22} - \sigma_{33})^2}$$

Substitute

$$\sigma_{ysAg} = \frac{1}{\sqrt{2}}\cdot\sqrt{\left[\sigma_{11} - \frac{\sigma_{11}\cdot\left(\nu_{Ag} - \nu_{St}\cdot\frac{E_{Ag}}{E_{St}}\right)}{1 - \nu_{Ag}}\right]^2 + \left[\sigma_{11} - \frac{\sigma_{11}\cdot\left(\nu_{Ag} - \nu_{St}\cdot\frac{E_{Ag}}{E_{St}}\right)}{1 - \nu_{Ag}}\right]^2}$$

$$\sigma_{ysAg} = \frac{1}{\sqrt{2}}\cdot\sqrt{2\cdot\sigma_{11}^{2}\left[1 - \frac{\left(\nu_{Ag} - \nu_{St}\cdot\frac{E_{Ag}}{E_{St}}\right)}{1 - \nu_{Ag}}\right]^2}$$

$$\sigma_{11} = \frac{\sigma_{ysAg}}{\left[1 - \frac{\left(\nu_{Ag} - \nu_{St}\cdot\frac{E_{Ag}}{E_{St}}\right)}{1 - \nu_{Ag}}\right]}$$

Substituting known values

$\sigma_{ysAg} := 140 \cdot 1e6$

$\nu_{Ag} := 0.367$

$\nu_{St} := 0.3$

$E_{Ag} := 30.3 \cdot 1e9$

$E_{St} := 200 \cdot 1e9$

$A := 1.1 \cdot 1.1$

$$\sigma_{11} := \frac{\sigma_{ysAg}}{\left[1 - \frac{\left(\nu_{Ag} - \nu_{St} \cdot \frac{E_{Ag}}{E_{St}}\right)}{1 - \nu_{Ag}}\right]}$$

$\sigma_{11} = 2.845 \times 10^{8}$

$P := \sigma_{11} \cdot A$

$\boxed{P = 3.443 \times 10^{8}}$ Newtons

If you had assumed that the strain in the 22 and 33 direction wa

$P := 403.12 10^{6}$

$\boxed{P = 4.031 \times 10^{8}}$ Newtons

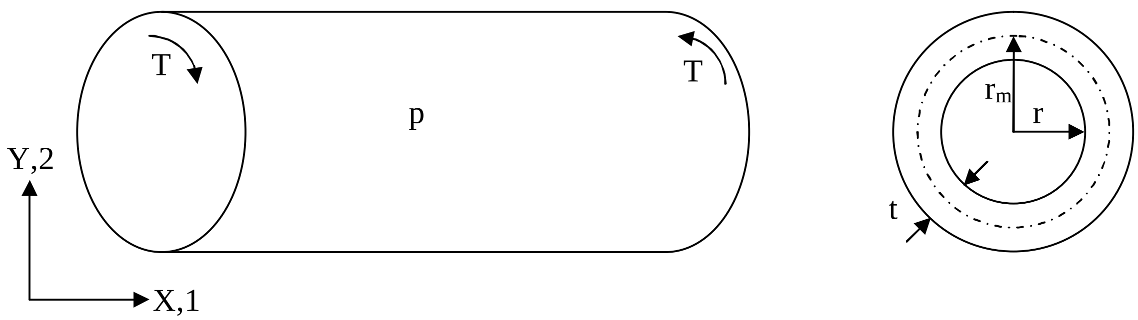
Therefore the Silver Braze yields before the Steel

Problem 3

The thin-walled cylinder, shown below, has an internal pressure of p = 800 kPa, and is subjected to a twist of T = 20 MNm. The inner radius of the cylinder is 2 m with a wall thickness of 20 mm. The shear stress on a thin-walled tube can be approximated by:

$$t = T/(2pr_m^2 t)$$

where T is the applied torsional moment, r_m is the radius to the median line, and t is the thickness of the cylinder. See Figure below:



a) Assuming that the ends have no effect on the stresses near the center of the cylinder,
   i. Determine the principal stresses
   ii. Determine the maximum shear stress

b) Check for failure by plastic yielding of the cylinder using the von Mises and Tresca criteria. Does the cylinder fail if the yield strength of the material used is 150 MPa?

c) If the cylinder is punctured to leave a tiny pinhole in the wall thickness, check if yielding will occur at the edge of the hole using the Tresca and von Mises criteria. The stress at the edge of the hole can be calculated using the principal stresses and the stress concentration factors at a hole in a pressurized cylinder. (Assume that the hole is small compared to the other relevant dimensions of the cylinder).

Solution 3 – 15 points

a) Find Shear due to Torsion

$MPa := 1 \cdot 10^6 \cdot Pa$

5

$r := 2 \cdot m \qquad t := 20 \cdot 10^{-3} m \qquad p := 800 \cdot 10^3 \cdot Pa$

$r_m := r + \frac{t}{2} \qquad r_m = 2.01 m$

$T := 20 \cdot 10^6 \cdot N \cdot m$

$\tau := \frac{T}{2 \cdot \pi \cdot r_m{}^2 \cdot t} \qquad \tau = 3.939 \times 10^7 \, Pa \qquad = 39 \text{ MPa}$

The shear on a unit element is negative

The stress due to pressure

$\sigma_1 := p \cdot \frac{r}{t} \qquad \sigma_2 := p \cdot \frac{r}{2 \cdot t}$

Note:
$\sigma_1 = \sigma_{\theta\theta}$
$\sigma_2 = \sigma_{zz}$

$\sigma_1 = 80 MPa \qquad \sigma_2 = 40 MPa$

Find the principal stresses

$\sigma_x := \sigma_2$

$\sigma_y := \sigma_1$

$\tau_{xy} := -\tau$

$$\sigma_{1p} := \frac{(\sigma_x + \sigma_y)}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \qquad \sigma_{1p} = 104.18 MPa$$

$$\sigma_{2p} := \frac{(\sigma_x + \sigma_y)}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \qquad \sigma_{2p} = 15.82 MPa$$

This can also be “calculated” graphically using Mohr’s circle

Additionally one can also calculate the third principle stress value, which would be equal to $s_{rr} \approx 0$

The maximum shear is:

$$\tau_{max} := \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$\tau_{max} = 44.18\text{MPa}$ When disregarding s_{rr}, or 52.09 MPa using s_{rr} as the lowest principal stress value

b) Check for Failure

5

By Tresca Criterion $k = \tau$

$\sigma_y := 150 \cdot \text{MPa}$

$k := \frac{\sigma_y}{2}$ $k = 75\text{MPa}$ $\tau_{max} = 44.18\text{MPa}$ or 52.09 MPa when using s_{rr} as the lowest principal stress value

As can be seen from the above values, $\tau_{max} < k$ Therefore the cylinder does not yie

By Von Mises Criteria

$\sigma_{11} := \sigma_2$ $\sigma_{22} := \sigma_1$ $\sigma_{33} := 0 \cdot \text{MPa}$ $\sigma_{12} := \tau_{xy}$ $\sigma_{31} := 0 \cdot \text{MPa}$ $\sigma_{23} := 0 \cdot \text{MPa}$

$$\sigma := \sqrt{\left(\frac{1}{2}\right) \cdot \left[\left(\sigma_{11} - \sigma_{22}\right)^2 + \left(\sigma_{22} - \sigma_{33}\right)^2 + \left(\sigma_{33} - \sigma_{11}\right)^2\right] + 3 \cdot \left(\sigma_{12}{}^2 + \sigma_{23}{}^2 + \sigma_{31}{}^2\right)}$$

$\sigma = 97.24\text{MPa}$ $s_y = 150\text{MPa}$

We can also check using the prinicpal stresses

$\sigma_{11} := \sigma_{1p}$ $\sigma_{22} := \sigma_{2p}$ $\sigma_{33} := 0 \cdot \text{MPa}$ $\sigma_{12} := 0 \cdot \text{MPa}$ $\sigma_{31} := 0 \cdot \text{MPa}$ $\sigma_{23} := 0 \cdot \text{MPa}$

$\sigma = 97.24\text{MPa}$ $\sigma_y = 150\text{MPa}$

According to Von Mises $\sigma < \sigma_y$ Therefore the cylinder does not yield

Cylinder does not yield

c) Hole in cylinder

5 The stress concentration factor and stresses on the hole can be calculated by using the principle stresses (S1 and S2, as calculated in Problem 3a) and a stress concentration calculation (see figure on next page).

$S_1 := \sigma_{1p} \qquad S_2 := \sigma_{2p}$

$\sigma_{A\theta\theta} := 3 \cdot S_2 - S_1 \qquad \sigma_{A\theta\theta} = -56.72\,\text{MPa}$

$\sigma_{B\theta\theta} := 3 \cdot S_1 - S_2 \qquad \sigma_{B\theta\theta} = 296.72\,\text{MPa}$

At point A

Tresca $\quad \dfrac{\sigma_{A\theta\theta}}{2} < \dfrac{\sigma_y}{2}$

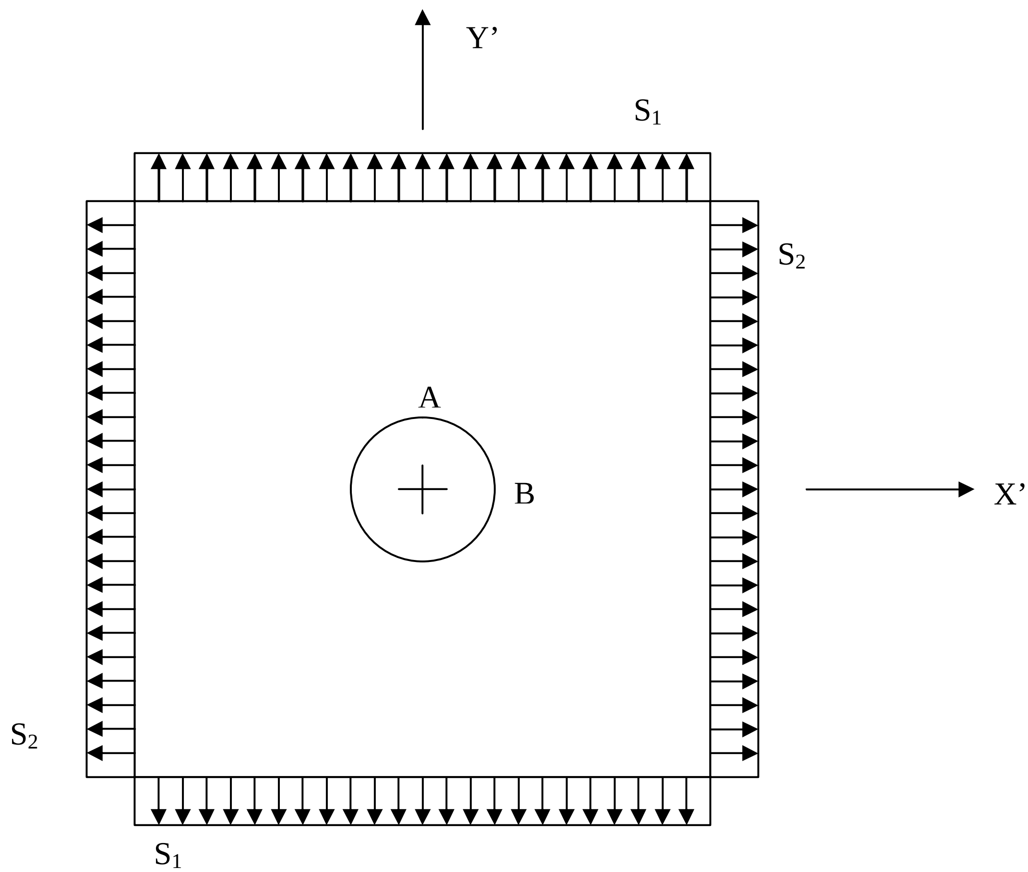
Von Mises $\quad \sigma < \sigma_y \qquad$ Therefore no yielding at point A

At Point B

Tresca $\quad \dfrac{\sigma_{B\theta\theta}}{2} > \dfrac{\sigma_y}{2}$

Von Mises $\quad \sigma > \sigma_y \qquad$ Therefore yielding at point B

Yielding Occurs at edge of hole



The Y’-X’ coordinate system is composed of the axis belonging to the coordinate system in a situation where no shear stresses are present (i.e. in the principal stress state).