Mathematics 55 — Professor K. A. Ribet
Last Midterm Exam — October 30, 1997

F295 Haas and 10 Evans
3:40–5:00 PM

Your Name: _____ TA: _____

Please check that you have all 6 pages of this exam booklet. Write your name on *each* page. This exam is 80 minutes long. At the conclusion of the exam, hand in this exam paper to your TA.

- This is a closed-book exam: no books, notes or calculators are allowed.
- You need not simplify your answers unless you are specifically asked to do so. In particular, it's fine for answers to contain binomial coefficients.
- It is essential to write legibly and *show your work*.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.

Problem	Maximum	Your Score
1	10	
2	8	
3	6	
4	11	
5	10	
Total	**45**	

Your Name: _____

1a *(5 points)***.** How many poker hands contain no ace, exactly one king, and at least one heart? (A poker hand contains five cards.)

1b *(5 points)***.** How many different strings of length ten can one make out of the letters in INDISCRETE?

Your Name: _____

2 *(8 points).* How many solutions to $x + y + z + w = 1097$ in non-negative integers x, y, z and w satisfy at least one of the inequalities $x \geq 100$, $y \geq 100$, $z \geq 100$? [Hint: use inclusion-exclusion.]

Your Name: _____

3 *(6 points)***.** In how many ways can a class of 15 be divided into 5 groups of 3 students in such a way that the two students named Ken are in the same group?

Your Name: _____

4a *(5 points)*. Persi's crooked penny comes up "heads" 2/3 of the time when it is tossed. What is the probability that exactly four heads come up when it is tossed six times?

4b *(6 points)*. A fair nickel and a fair penny are tossed simultaneously until both come up "tails." What is the expected number of tosses? (One "toss" is a flip of the two coins.)

Your Name: _____

5a *(4 points).* What is the coefficient of $x^{30}y^{29}$ in the expansion of $(x-2y)^{59}$?

5b *(6 points).* How many integers are needed to guarantee that two of them leave the same remainders on division both by 15 and by 21? [Hint: note that 15 and 21 have a common factor.]