Mathematics 55 — Professor K. A. Ribet
First Midterm Exam — September 25, 1997

F295 Haas and 1 Leconte
3:40–5:00 PM

Your Name: ______________________________ TA: ________________________

Please check that you have all 7 pages of this exam booklet. Write your name on *each* page. As you turn through the pages, look for the easy questions—do them first. This exam is 80 minutes long.

- This is a closed-book exam: no books, notes or calculators are allowed.
- You need not simplify your answers unless you are specifically asked to do so.
- It is essential to write legibly and *show your work*.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.

Problem	Maximum	Your Score
1	10	
2a–b	10	
2c and 3	11	
4	10	
5	8	
6	11	
Total	**60**	

At the conclusion of the exam, hand in this exam paper to your TA.

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1a *(4 points)***.** Find the remainder when 2^{55} is divided by the prime number 53.

1b *(6 points)***.** Suppose that f and g are functions $S \to S$, where S is the set of positive integers less than 10^3. If the composition $f \circ g$ is 1-1 and onto, must f and g be 1-1 and onto? (Give a short proof or a counterexample.)

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In the following problems, it may be useful to know that $203 \cdot 83 - 39 \cdot 432 = 1$.

2a *(5 points)***.** Find an integer x such that $83x \equiv 1 \bmod 432$.

2b *(5 points)***.** Find an integer y such that $39y \equiv 4 \bmod 203$.

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2c *(6 points)*. Find an integer z such that $z \equiv 2 \mod 39$ and $z \equiv 3 \mod 203$.

3 *(5 points)*. Can you conclude that $A = B$ if A, B and C are sets such that $A \cap C = B \cap C$ and $A \cup C = B \cup C$? (Explain why or why not.)

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4 *(10 points)***.** Numbers A_n are defined as follows:

$$A_0 = 0; \quad A_1 = 1; \quad A_n = 5A_{n-1} - 6A_{n-2} \text{ for } n \geq 2.$$

Prove that $A_n = 3^n - 2^n$ for all $n \geq 0$.

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5 *(8 points)*. Suppose that $f(x) = 5^x$ and $g(x) = 10^x$. Decide whether each of the following statements is true. (Logarithms are to the base e.)

(a) $f(x) = O\big(g(x)\big)$.
(b) $g(x) = O\big(f(x)\big)$.
(c) $\log g(x) = O\big(\log f(x)\big)$.
(d) $f(x) = O\big(\log g(x)\big)$.

Explain your reasoning.

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6a *(6 points)***.** Find an integer d such that $(M^{11})^d \equiv M \bmod 55$ for all integers M such that $\gcd(M, 55) = 1$.

6b *(5 points)***.** Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology.