George M. Bergman   Fall 1996, Math 53   25 Sept., 1996

10 Evans Hall   **First Midterm**   10:10-11:00 AM

110 Barrows Hall (sections 101, 102, 103)

525 McCone Hall (sections 107, 109)

1. (25 points) Let C be the curve given by the parametric equations $x = e^t$, $y = e^{2t}$, $(-\infty < t < +\infty)$.

(a) (13 points) Describe C by an equation relating x and y, and restrictions on the values of x if necessary, and sketch the curve. (Hint: given e^t, think of how you could find e^{2t}.)

(b) (12 points) Find an equation for the line that is tangent to C at the point having parameter t. (This can be done with or without the help of part (a).)

2. (25 points) Let f be a continuous positive real-valued function on the interval $[0, \pi]$. Let C denote the curve whose formula in polar coordinates is $r = f(\theta)$ $(0 \le \theta \le \pi)$.

Obtain a formula for the area of the surface of revolution of C about the polar axis (the line $\theta = 0$, i.e., what in Cartesian coordinates is called the x-axis). This formula is not in Stewart, but you should be able to derive it from formulas that were given, which you may assume. Show your work, or at least briefly sketch how you got your answer.

3. (25 points) Calculate the arc-length of the space curve

$$\mathbf{r}(t) = (2t^3/3 + 1/3, \ t + 7, \ t^2 + 1) \text{ for } -1 \le t \le 2.$$

4. (25 points) Suppose $f(x, y)$ is a real-valued function of two variables, defined for all real numbers x and y.

(a) (7 points) Given a point (x_0, y_0), and a real number L, define what it means for $\lim_{(x, y) \to (x_0, y_0)} f(x, y) = L$ to hold.

(b) (7 points) Define what it means for f to be continuous at (x_0, y_0).

(c) (11 points) Let f be the function defined by the formulas $f(x, y) = 0$ if $x \ne y^2$, $f(x, y) = x$ if $x = y^2$. At what points is f continuous, and at what points is it discontinuous? Give some explanation for your conclusion as to whether f is or is not continuous at $(0, 0)$. For other points, it suffices to describe at which f is, and at which it is not continuous.