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Fall 1996, Math 53
Second Midterm

28 Oct., 1996
10:10-11:00 AM

1. (33 points) (a) (20 points) Find the maximum and minimum values of the function $(x+1)^2 + y^2$ on the ellipse $x^2 + y^2/4 = 1$, and say at what points these values occur.
- (b) (13 points) Give the gradients $\nabla((x+1)^2 + y^2)$ and $\nabla(x^2 + y^2/4)$ at one point of the ellipse where $(x+1)^2 + y^2$ assumes its maximum value, and at one point of the ellipse where it assumes its minimum value.
2. (33 points) (a) (13 points) Let D be the triangle bounded by the lines $x = 1$, $y = 2$, and $2x + y = 2$. Give the two expressions for the double integral $\iint e^{x-y} dA$ as an iterated integral (i.e., the expressions using the two orders of integration).
- (b) (20 points) Evaluate the above double integral (by any method).
3. (34 points) Suppose f and g are twice-differentiable functions of a real variable (i.e., assume their second derivatives, f'' and g'' , exist), and suppose we define a function E of two variables by the equation
- (a) (12 points) Express the first and second partial derivatives E_x , E_y , E_{xx} , E_{xy} , E_{yx} , and E_{yy} in terms of f and g .
- (b) (6 points) Given a point (x_0, y_0) , under what conditions on f and g will (x_0, y_0) be a critical point of the function E ?
- (c) (16 points) If (x_0, y_0) is a critical point of E , find conditions on f and g for this point to be a local maximum, conditions for it to be a local minimum, and conditions for it to be a saddle point. Also state the conditions under which the criteria we have learned *do not determine* the nature of the critical point.