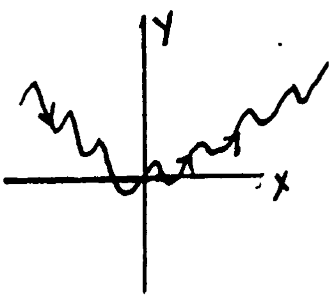
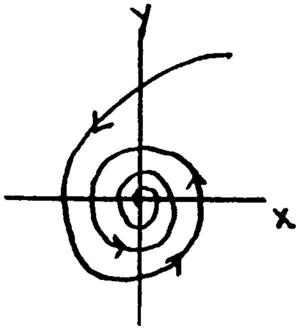
Mathematics 53
Midterm #1 , 27 Sept 2000
J.Wagoner

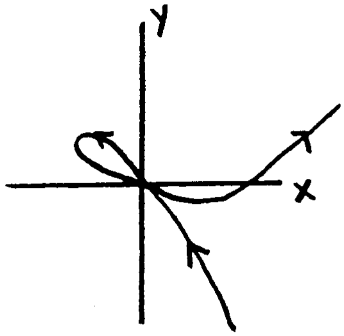
Introduction Write your name, GSI's name (5 points), and section number (5 points) on your Blue Book (5 points) right now. Partial credit may be given if your work justifies it. Best wishes on the exam.

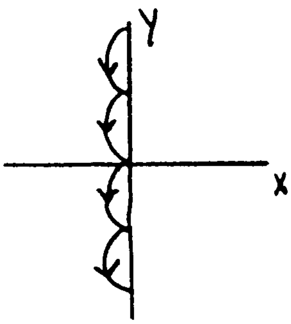
Problem #1 From the diagrams below select the picture which best represents each of the following parametrized curves.

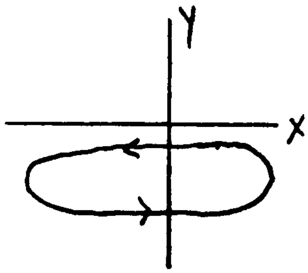
(A)	$a(t) = (cos(t) - 1, sin(t) - t)$	(B)	$b(t) = (3cos(t) + 1, sin(t) - 2)$
(C)	$c(t) = (t(t-1), t(t-1)(t-2))$	(D)	$d(t) = (cos(3t), sin(2t))$
(E)	$c(t) = (e^t cos(t), e^t sin(t))$	(F)	$f(t) = (t, t^2 + sin(t))$

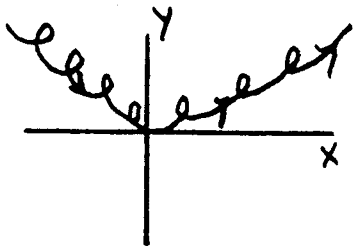
(1) 

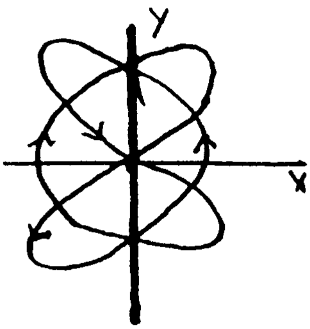
(2) 

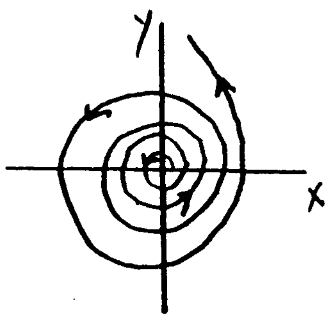
(3) 

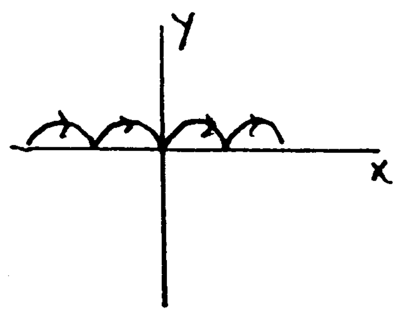
(4) 

(5) 

(6) 

(7) 

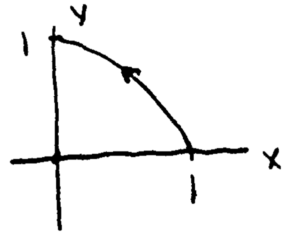
(8) 

(9) 

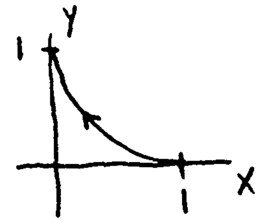
Problem #2 Consider the curve C defined by $r(t) = (cos^3(t), sin^3(t))$ for $0 \leq t \leq 2\pi$.

(A) Which of the following two pictures best represents the part of the curve [redacted] for $0 \leq t \leq \pi/2$. Give a reason for your answer. Hint: Consider $t = \pi/4$.

(1)



(2)



(B) Write down the formula for the length L of the curve C and use it to compute L.

Problem #3 (A) Sketch the graph of $r = 3sin(2\theta)$ where $0 \leq \theta \leq 2\pi$.

(B) Use small x's to mark the portion of the curve corresponding to $6\pi/4 \leq \theta \leq 7\pi/4$.

Problem #4 (A) Sketch the graph of $r = f(\theta) = 2/sin(\theta)$ where $\pi/4 \leq \theta \leq \pi/2$.

(B) Sketch the region R consisting of those points (r, θ) where $0 \leq r \leq f(\theta)$.

(C) Compute the area of R.

Problem #5 Consider the parametrized curve $r(t) = (t + cos(t), 2t + sin(t))$ where $-\infty < t < \infty$.

(A) For which values of t does the curve have a horizontal tangent? Explain.

(B) For which values of t does the curve have a vertical tangent? Explain.

Problem #6 Let P be the plane containing the three points $A = (7, -3, -1)$, $B = (1, 0, 2)$, and $C = (-1, -2, 6)$.

(A) Find a vector normal to the plane P and having length equal to 1.

(B) Find the linear equation of the plane P.

Problem #7 In R^2 find the orthogonal projection of $v = (3, 4)$ onto the line L which is perpendicular to $2x - y - 1 = 0$ and which passes through the origin O = (0,0).

Problem #8 Compute the area A of the parallelogram determined by the vectors $A = (3, 2)$ and $B = (2, 1)$.