MATH 185 – MIDTERM #2 Spr 02

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INSTRUCTIONS: Answer each question on a separate sheet of paper, and write your name on each page. Each problem counts equally.

Problem #1. Find the residue at $z_0 = 0$ of

$$z^8 \sin\left(\frac{1}{z}\right).$$

Problem #2. Compute the integral

$$\int_C \frac{1}{z^8+1}\,dz,$$

where C denotes the positively oriented circle of radius 4 about 0.

Problem #3. Calculate

$$\int_0^\infty \frac{1}{(x^2+1)^3}\,dx.$$

Problem #4. Expand the function

$$f(z) = \frac{1}{z(z+2)^2}$$

into powers of z, in both the regions $|z| < 2$ and $|z| > 2$.

Problem #5. (i) Let f be an entire function. State the formula representing $f^{(n)}(z)$ as a contour integral and then prove *Cauchy's inequality*

$$|f^{(n)}(z_0)| \leq \frac{n! M_R}{R^n},$$

where M_R denotes the maximum of $|f|$ on the circle $|z - z_0| = R$.

(ii) State and prove *Liouville's Theorem.*

Problem #6. Assume f is analytic within the disk $|z| < R$. Use Cauchy's integral formula

$$f(z) = \frac{1}{2\pi i}\int_C \frac{f(w)}{w-z}\,dw$$

for an appropriate contour C to show that we can write

$$f(z) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!} z^n.$$

(You do not need to show rigorously that this series converges for $|z| < R$.)