(2) Grade by subtraction — so if final result is wrong owing to one step, deduct for that step only. do not deduct for result & step together.

(1) Please mark each page, blank or not, to show you have read it

TEST: 156
MEAN [redacted]
SD 43 [redacted]
S.M.

260

University of California, Berkeley
Mechanical Engineering
ME106 Fluid Mechanics
1st Test, S06 *Prof S. Morris*

NAME SOLUTIONS

1. (65) The sketch shows a siphon of uniform diameter being used to drain a tank.

(15) (a) Sketch the streamlines qualitatively.

(25) (b) Find the speed V at the exit of the siphon.

(25) (c) Find the pressure at the inlet of the siphon (it is not hydrostatic), assuming that the flow there is uniform across the siphon.

(3) Please give mean & SD on each question & for test as a whole

Show subtotals

NOTE

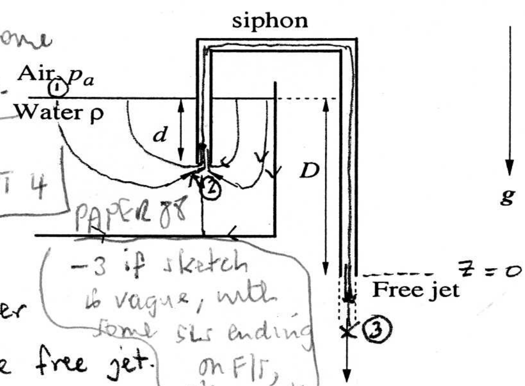
[redacted] } S.M.

Q1: 53 MEAN; SD 13

Deduct (3) from this if only some SLs are shown as going to F.S.

see solutions to HWK SET 4

PAPER 88



(15) (a) All streamlines begin on the air-water interface, enter the siphon and leave in the free jet.

-3 if sketch is vague, with some SLs ending on F.S., others not

$z = 0$ at siphon exit.

1
2
3

o Streamtubes cannot end on walls in incompressible flow — reader p 8-5

Streamline 1-2-3 is used with the Bernoulli equation

-5 Clearly defined origin with consistent use of z coord

(25) (b) BE 1-3

Correctly stated -8

$$\underbrace{\tfrac{1}{2}\rho V_1^2 + p_1 + \rho g D}_{\text{at } ①} = \tfrac{1}{2}\rho V_3^2 + p_3 + \rho g(0)$$

$$\underbrace{p_1 = p_3 = \text{atmospheric}}_{-5} \; ; \; V_3 = V$$

$$V^2 = 2gD + V_1^2$$

For analysis with correct underlying principles but inconsistent working give ... see #123

Because the tank is shown as being large, and because the depth d to the inlet is shown as being large compared with the siphon diameter $V_1 \ll V$ -5

$$\Rightarrow \quad V^2 = 2gD \qquad \boxed{V = \sqrt{2gD}}$$ -2

Torricelli's theorem (again

If they are completely lost and would otherwise get no points, give them credit (8) for BE, as on paper

Give the hopeless ones something — see

SOLUTIONS

Blank page

25

(c) Either (i) Because the siphon diameter is constant, speed is constant within the siphon.

$\boxed{\text{BE } 2\text{-}3}$ −5

$$\frac{1}{2}\rho V_2^2 + p_2 + \rho g z_2 = \frac{1}{2}\rho V_3^2 + p_3 + \rho g(0)$$

−5 −5 −5

$\boxed{z_2 = D-d}$, $\boxed{V_2 = V_3}$, $\boxed{p_3 = p_a}$ (free jet)

$$\Rightarrow \boxed{p_2 = p_a - \rho g(D-d)} \quad -5$$

(Because there is no acceleration along the streamline from 2-3, p is hydrostatic within the siphon.)

or (ii) $\boxed{\text{BE } 1\text{-}2}$ −5

$$\frac{1}{2}\rho V_1^2 + p_1 + \rho g D = \frac{1}{2}\rho V_2^2 + p_2 + \rho g(D-d)$$

−5

$\underbrace{V_1^2 \ll V_2^2 = 2gD}_{-10}$, $\boxed{p_1 = p_a}$

$$\Rightarrow 0 + p_a + \rho g D = \rho g D + p_2 + \rho g(D-d)$$

$$\boxed{p_2 = p_a - \rho g(D-d)} \quad -5$$

(Equivalent arguments)

MEAN. [redacted]51 } SM
SD [redacted]21

SOLUTIONS

2. (65) In a certain flow, the velocity field given by $\mathbf{V} = \frac{1}{2}cr\,\hat{\mathbf{r}} - cz\,\hat{\mathbf{k}}$, where c is a positive constant, and r, θ, z are cylindrical polar coordinates.

35 **(a)** Find the acceleration $\mathbf{a}$ as a function of position $\mathbf{r}$.

30 **(b)** Show that the velocity field $\mathbf{V}$ satisfies Euler's equation of motion.

Given.

In cylindrical polar coordinates r, θ, z, if the velocity field $\mathbf{V} = v_r\hat{\mathbf{r}} + v_\theta\hat{\theta} + v_z\hat{\mathbf{k}}$, and $\mathbf{F} = f_r\hat{\mathbf{r}} + f_\theta\hat{\theta} + f_z\hat{\mathbf{k}}$ is an arbitrary vector, then

$$\nabla \times \mathbf{F} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & r\hat{\theta} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial z} \\ f_r & rf_\theta & f_z \end{vmatrix}, \qquad \frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} = \left(\frac{\mathrm{d}v_r}{\mathrm{d}t} - \frac{{v_\theta}^2}{r}\right)\hat{\mathbf{r}} + \left(\frac{\mathrm{d}v_\theta}{\mathrm{d}t} + \frac{v_r v_\theta}{r}\right)\hat{\theta} + \frac{\mathrm{d}v_z}{\mathrm{d}t}\hat{\mathbf{k}}.$$

Read their working — see PAPER 63 for a case where the answer is correct but the working is gibberish

(a) Here $v_r = \frac{1}{2}cr$, $v_z = -cz$, $v_\theta = 0$

wrong $\frac{d}{dt} = \frac{\partial}{\partial t} + \underline{u}\cdot\nabla(\,)$ (-10) (but only deduct once!)

$$\frac{d}{dt} = \underbrace{\frac{\partial}{\partial t}}_{=0,\ \text{const}} + v_r\frac{\partial}{\partial r} + \underbrace{\frac{v_\theta}{r}\frac{\partial}{\partial\theta}}_{=0,\ v_\theta = 0 = \frac{\partial}{\partial\theta}} + v_z\frac{\partial}{\partial z} = \frac{1}{2}cr\frac{\partial}{\partial r} - cz\frac{\partial}{\partial z}$$

Give 20/35 for answers that are correctly worked, but where which is wrong stop well short of the final answer for a

$$(\underset{\sim}{a})_r = \frac{dv_r}{dt} - \frac{\overset{0}{v_\theta^2}}{r} = \left(\frac{1}{2}cr\frac{\partial}{\partial r} - cz\frac{\partial}{\partial z}\right)\frac{1}{2}cr = \frac{1}{4}c^2r \quad \because \frac{\partial r}{\partial z} = 0$$

(+5)

$$(\underset{\sim}{a})_\theta = \frac{dv_\theta}{dt} + \frac{v_r v_\theta}{r} = 0 \qquad (v_\theta \equiv 0)$$

(+1) sign

$$(\underset{\sim}{a})_z = \frac{dv_z}{dt} = \left(\frac{1}{2}cr\frac{\partial}{\partial r} - cz\frac{\partial}{\partial z}\right)(-cz) = c^2z \quad \because \frac{\partial z}{\partial r} = 0$$

~~+5~~ (-5): small math error or not recognizing $\frac{dr}{dt} = v_r$, $\frac{dz}{dt} = v_z$

PAPER 83

5/35 if reach $\underset{\sim}{a} = \frac{d}{dt}(v_r)\hat{r} + \frac{d}{dt}(v_z)\hat{k}$ but don't evaluate

$$\Rightarrow \boxed{\underset{\sim}{a} = \frac{1}{4}c^2r\,\hat{r} + c^2z\,\hat{k}}$$

-3 w/o unit vectors

PAPER 48 solution

• If the solution is basically correct but there is insane steps leading to an incorrect answer give 25/35

(b) $\underline{V}$ satisfies Euler's EOM for uniform density flow if and only if $\nabla\times\underline{a} = 0$.

30/30

$$\text{Here}\quad \nabla\times\underline{a} = \frac{1}{r}\begin{vmatrix} \hat{r} & r\hat{\theta} & \hat{k} \\ \frac{\partial}{\partial r} & 0 & \frac{\partial}{\partial z} \\ \frac{1}{4}c^2r & 0 & c^2z \end{vmatrix} = \underset{\sim}{0} \quad \text{since} \quad \underbrace{\frac{\partial z}{\partial r}}_{(z\ \text{fixed})} = 0 = \underbrace{\frac{\partial r}{\partial z}}_{(r\ \text{fixed})} \Rightarrow \boxed{\nabla\times\underline{a} = 0}$$

QED //

If correct idea & evaluation but $\underline{a}$ is wrong from part (a) see PAPER 88

take $\nabla\times V$ w/o explanation 0/30

20/30 never plug into $\nabla\times\underline{a}$.

5/30 for $\nabla\times\underline{a}$ only.

-5: (massively) incorrect expansion of determinant...

(-10) wrong formula for $\nabla\times\underline{a}$ ...i.e. $\frac{1}{r}\left(\frac{\partial(ra_\theta)}{\partial r} - \frac{\partial a_r}{\partial\theta}\right)\hat{k}$ see #94

If they forget to calculate one component of $\underline{a}$, but the other is done correctly 60/65

SOLUTIONS

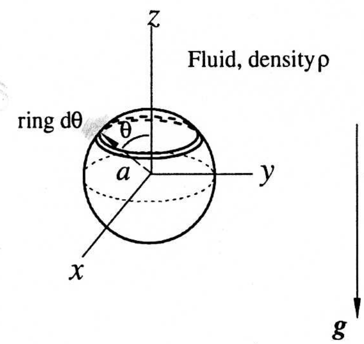
3. **(70)** The figure shows a sphere of radius a that is completely submerged in a stationary fluid. Find the resultant vertical component of force exerted by the fluid on the sphere in the following two ways:

20 **(a)** by using Archimedes's principle; and

50 **(b)** by integrating the vertical component of pressure force over the surface of the sphere.

Given. (a) The hydrostatic pressure on the surface of the sphere is given as a function of the co-latitude θ by $p = p_0 - \rho g a \cos\theta$ where p_0 is the pressure at the equatorial plane $z = 0$ of the sphere. (b) $\int_0^\pi \sin\theta \cos\theta \, d\theta = 0$, and $\int_0^\pi \cos^2\theta \sin\theta \, d\theta = 2/3$.

MEAN: 52[redacted]
SD: 21[redacted]) SM

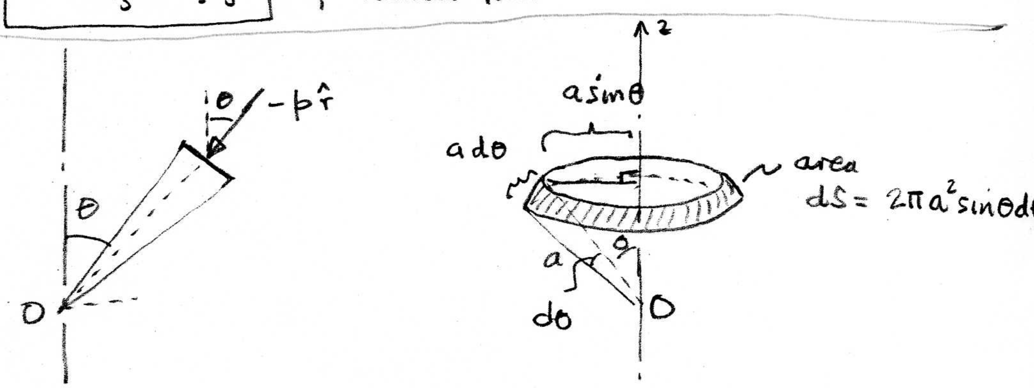


(a) ARCHIMEDES: a body completely submerged in a fluid at rest experiences a vertical force equal to the weight of fluid displaced.

Here ρ uniform, displaced volume $= \frac{4}{3}\pi a^3$

$\Rightarrow$ $\boxed{F = \frac{4}{3}\pi a^3 \rho g}$, vertical force exerted by fluid on sphere

(b)



The vertical component of stress is $-p\cos\theta$

Because this is independent of the azimuthal angle, the total force

+15
10 Approach of integrating distrib. force over surface

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exerted on the ring dS $= \underbrace{- p\cos\theta\, dS}_{+15}$

The total force exerted by the fluid on the sphere is given by

$$F = -\int p\cos\theta\, dS$$

$$= -2\pi a^2 \int_0^{\pi} p\cos\theta\sin\theta\, d\theta \qquad \text{since } \underbrace{dS = 2\pi a^2 \sin\theta\, d\theta}_{+15}$$

But $p = p_0 - \rho g a\cos\theta$, so

$$F = -2\pi a^2 p_0 \underbrace{\int_0^{\pi} \cos\theta\sin\theta\, d\theta}_{=0} + 2\pi\rho g a^3 \underbrace{\int_0^{\pi}\cos^2\theta\sin\theta\, d\theta}_{2/3}$$

calculation +5

(uniform pressure acting
over a closed surface exerts
zero resultant force on the surface

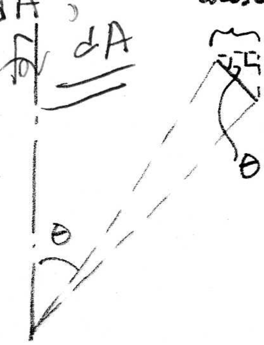
-1 for extremely trivial slips, as on PAPER 13 $\frac{3}{5}$0

$$\boxed{F = \tfrac{4}{3}\pi\rho g a^3}$$

same as part (a).

(IF final result is wrong eg because of incorrect dA, deduct only for $\underline{\underline{dA}}$

a cosθ dθ



NOTE This can also be solved using local stress equilib

Area of the ring projected onto $z = 0$ is

$$(2\pi a^2 \sin\theta\, d\theta)\cos\theta = dS\cos\theta$$

Then $\boxed{F = -\int p\cos\theta\, dS}$

END

The $\cos\theta$ refers to the $\underline{k}$ component of the directed area $dS\,\hat{r}$.
Pressure p is a $\underline{\text{scalar}}$.