UNIVERSITY OF CALIFORNIA
COLLEGE OF ENGINEERING

E77: INTRODUCTION TO COMPUTER PROGRAMMING FOR SCIENTISTS AND ENGINEERS

Spring 2006
Second Midterm Exam—April 12, 2006

[45 points = 45 minutes]

Question	Points	Grade
A	15	
B	15	
C	8	
D	3	
E	4	
TOTAL	**45**	

Notes:

1. Write your name below and on the top right corner of every page.
2. Please give all your answers only in the spaces provided.
3. You may NOT ask any questions during the exam.
4. You may NOT leave the exam room before the exam ends.

Your **PRINTED NAME** + **signature**: ____

Your **E77** LECTURE SECTION **1** or **2** (Circle your section #)

Circle your Lab Section (where the graded midterms will be returned).

#11: MW 8-10 (Etch)	#12: MW 10-12 (Etch)	#13: MW 2-4 (Etch)	#14: MW 4-6 (Etch)
#15: TuTh 8-10 (Etch)	#16: TuTh 10-12 (Etch)	#17: TuTh 12-2 (Etch)	#18: TuTh 2-4 (Etch)
#19: TuTh 4-6 (Etch)	#20: MW 8-10 (Latimer)	#21: TuTh 8-10 (Latimer)	

A. Linear Algebra (<u>15 points</u>)

1. **(+3)**

$$-4x+5y=10$$
$$12x-15y=8$$

What is the determinant of the coefficient matrix of these equations?

$$A=\begin{pmatrix} \_ & \_ \\ \_ & \_ \end{pmatrix},\quad \det(A)=\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

A= [-4,5;12,-15]
det(A) = 0

Does this set of equations have a unique solution?

YES NO

NO

2. **(+4)** For what value of a will the following set have a solution in which both x and y are nonzero? Find the relationship between x and y

$$4x-ay=0$$
$$-3x+6y=0$$

$a=$ ______, $x=$ ___________, $y=$ ___________

a=8; x=2*y; y=0.5*x

3. **(+4)**

   a. **(+0.5)** What MATLAB function calculates the determinant of a matrix A?

_____det(A)____

   b. **(+1)** Does the matrix A above have to be square? (Circle one choice)

   YES NO
   YES

   c. **(+1)** What does the backslash (\) operator do:

1) For an *n*-by-*n* system of linear equations?

_Solves for a linear equation defined by A*X = B; X=A\B

2) For an *m*-by-*n* (*m*>*n*) system of linear equations?

_Solves using least squares the over-determined system: A*X=B; X=A\B

d. **(+1)** What MATLAB function calculates the inverse of a matrix?

____________inv()____

e. **(+0.5)** Does the matrix to be inverted have to be square? (Circle one choice)

YES NO

YES

4. **(+4)** Write a single MATLAB statement to solve the system of equations:

$$A*(x.*x) = b$$

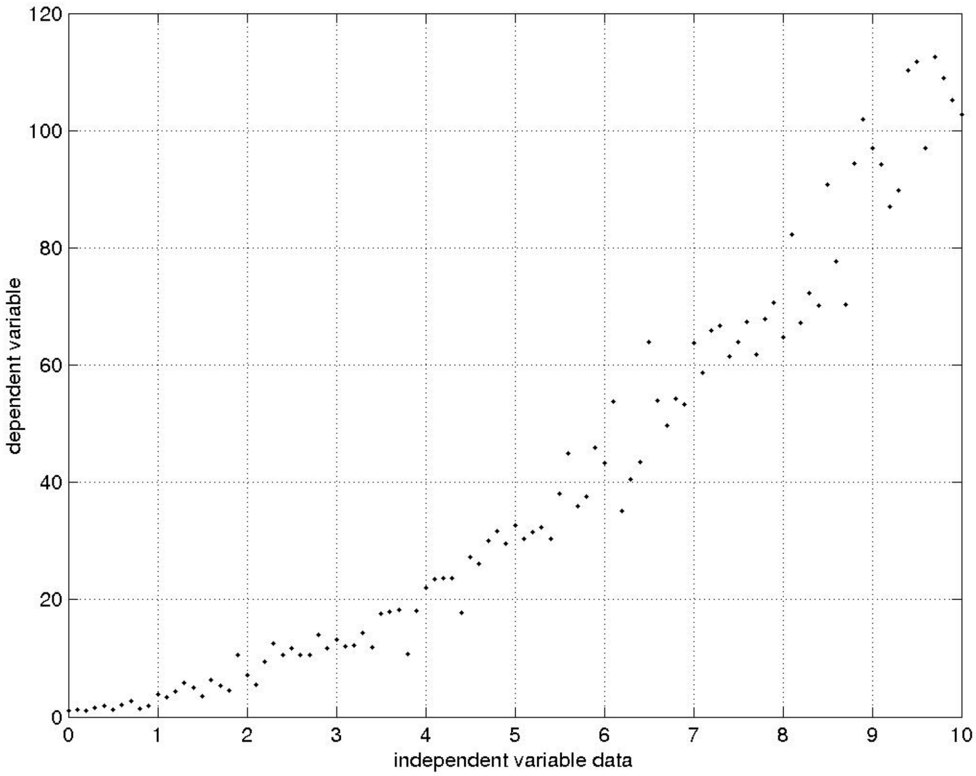
where *A* is a square matrix of rank n, *x* and *b* are vectors of size $n \times 1$. *A* and *b* are given.

Find *x*:

____*x=(A\b).^(0.5)* or *x=sqrt(A\b)*______________________________

B. Least Squares (<u>15 points</u>)

1. **(+7 1pt. each)**



a. Given the above data the best fit to try is a:

a) Linear least squares fit
b) Quadratic least squares fit
c) Weighted linear least squares fit
d) Weighted quadratic least squares fit – this is correct

b. When $a = \begin{bmatrix} a_1 \\ a_0 \end{bmatrix} = \left(X^T X\right)^{-1} X^T Y$ where a_0 is the Y-intercept of the least squares line and a_1 is the slope of the line which of the following is the right choice.

a) $X = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$

$\boxed{\text{b) } X = \begin{bmatrix} x_1 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{bmatrix}}$

c) $X = \begin{bmatrix} 1 & 1 \\ \vdots & \vdots \\ 1 & 1 \end{bmatrix}$

d) $X = \begin{bmatrix} x_1 & x_1 \\ \vdots & \vdots \\ x_n & x_n \end{bmatrix}$

c. When fitting a least squares quadratic $a_2x^2 + a_1x + a_0$ to data $(x_1, y_1)\ldots(x_n, y_n)$ using the normal equation $a = \begin{bmatrix} a_2 \\ a_1 \\ a_0 \end{bmatrix} = \left(X^T X\right)^{-1} X^T Y$ what is X ?

____ $X = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ \ldots & \ldots & \ldots \\ x_n^2 & x_n & 1 \end{bmatrix}$ ______________________________

d. In a least squares fit the sum of the residuals is:

a) zero – this is correct b) one

c) positive d) negative

e. Consider the program

```
[P, S]=polyfit(xdata,ydata,1);
yls=polyval(P,xdata);
ylsbar=mean(yls);
ydatabar=mean(ydata);
```

Then ylsbar-ydatabar is:

a) always positive b) always negative

c) always zero - this is correct d)none of the above

f. MATLAB polyfit fits a:

a) least squares line b) least squares quadratic

c) least absolute deviation line

d) least squares polynomial of specified degree – this is correct

g. The MATLAB command to compute the value of a polynomial given its coefficient is:

a) polyfit b) polyval – this is correct

c) polytope d) polygon

2. **(+2)** Write a single MATLAB statement for 10 uniform random numbers between 0 and π

`Ru10 =` _______pi.*rand(10,1) or pi*rand(1,10)__or pi*rand(10,1) or pi.*rand(1,10);

3. **(+3)** Suppose that you want to define 50 numbers from a normal random distribution with the `mean = 0.5` and the standard deviation `std = 0.2`. Write a single MATLAB statement:

`Rn50 =`_____mean+(std*randn(50,1))____;

4. **(+3)** Given vectors of increasing `x` and the corresponding measurements, `y`, write two MATLAB statements to evaluate the quadratic least squares fit of `y`, call it `p`, and evaluate this fit for all values of `x`, creating a new vector `yLS`. (Hint: Use two specific MATLAB functions.)

`p =` _____polyfit(x,y,2)____;

`yLS =` _____________polyval(p,x)____;

C. Root Finding (8 points)

1. **(+1.0)** In the bisection method for finding roots, the value of the function at the endpoints of the starting interval must be:

a) Positive

b) Negative

c) Positive at the first endpoint and negative at the second

d) Positive at one endpoint and negative at the other – this is correct

2. **(+2.0)** To find the roots of the function $y = (x-1)^2$ one should use the:

a) Bisection method on the interval [-2,2]

b) The Newton method but the initial value has to be 2

c) The Newton method with any initial value – this is correct

d) none of the above

3. **(+2.0)** The roots of the function $y = x+1$

a) will be found by the bisection method if the initial endpoints are 0 and 1

b) will be found by the bisection method in one step if the initial endpoints are -2 and 0 – this is correct

c) cannot be found by the bisection method

4. **(+1.0)** The MATLAB function fzero used on the function y = tan (x) with initial interval [1, 2] will:

a) result in a stack overflow error

b) find a true root

c) declare it has found a root but make a mistake - this is correct

d) return the answer x = 0

5. **(+2.0)** When the Newton method is initialized at a minimum of a function to find its roots:

a) it converges to the root extremely fast

b) it fails to converge – this is correct

c) it converges to the root but slowly

d) none of the above

D. Differentiation and Integration (3 points)

1. **(+1.0)** Consider the function: $y = x^2$. Let x = [0:0.1:10] and y = polyval([1 0 0], x). A 2nd order centered difference method is used to estimate the derivative of the function at the points [0.05:0.1:9.95]. The error in the estimates

   a) will grow from 0.05 to 9.95
   b) will be zero at all points

   c) will be zero at all points except the first and last point – this is correct

   d) is impossible to predict

2. **(+1.0)** The most appropriate numerical differentiation method to compute the derivative of tan(x) at x = pi/2 is:

   a) 2nd order centered difference

   b) 4th order centered difference

   c) 6th order centered difference

   d) none of the above – this is correct

3. **(+1.0)** Examine the program below. What will be the output for disguise(@myFunc, 0.1, 0, 1)?

```
function answer = disguise(func,h,l,u)
x = [l:h:u];
for c = 1:length(x),
    y(c) = feval(func, x(c));
end
answer = 0;
for c = 1:(length(x)-1)
    answer = answer + h*(y(c)+y(c+1))/2 ;
end
```

where:

```
function y = myFunc(x)
y=x;
```

answer=____________0.5______________________________

E. Representation of Numbers (<u>4 points</u>)

The IEEE Double Precision representation of a number has the form

```
S EEEEEEEEEEE FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
0 1        11 12                                                63
```

The bias in the exponent field is 1023.

1. (+2.0) Give an expression for the largest binary number it can represent.

Ans ____0 1111…10 111….1111_or 2^{1024}x1.11111….111 or 2^{1024}x2
Or 2^{1024}x(1+1/2 + …..+ 1/ 2^{51})

2. **(+1.0**) Give an expression for the smallest binary number it can represent.

Ans ____ 0 000….01 000….0 or
0 000….00 000….01 or
1 111….10 111….11

3. **(+1.0**) Give an expression for the smallest decimal number it can represent.

Ans _____4.94x10^{-324} ______