

Math 128A Midterm 1, 21 Feb 2003. J. Strain

1. Round  $\pi = 3.14159\dots$  to 4 decimal digits. Evaluate the absolute and relative errors and the number of significant digits. Repeat with  $\pi \times 10^{200}$ .

2. Consider the iteration  $x_0 = 1/2$ ,

$$x_{n+1} = x_n (2 - \pi x_n).$$

Evaluate  $\lim_{n \rightarrow \infty} x_n$ . Find the rate of convergence to the limit as  $n \rightarrow \infty$ .

3. Construct the quadratic polynomial  $P(x)$  which interpolates

$$f(x) = \sin \frac{\pi x}{2} \quad \text{at } x = -1, 0, 1.$$

Show that

$$|f(x) - P(x)| \leq \frac{\pi^3}{72\sqrt{3}} \quad \text{for } |x| \leq 1.$$