M. Rieffel Math 128A October 14, 1997
First Midterm Exam

SHOW YOUR WORK COMPLETELY AND NEATLY. Total points = 60.

1. a) For a function f and distinct points a, b, and c, define what is
3 meant by $f[a, b, c]$.

b) Find the Lagrange form of the polynomial p which
5 interpolates $f(x) = 4x/(x+1)$ at 0, 1, and 3.

5 c) From your answer to part b) find $f[0, 1, 3]$.

2. a) Define precisely what it means for a convergent sequence of
3 numbers to converge linearly.

b) View $g(x) = x/(x+2)$ as an iteration function. Note that 0
8 is a fixed point. Prove that for any initial guess which is > 0
the resulting iteration sequence will converge linearly to 0.

3. a) State the simple midpoint rule for numerical integration, and indicate
3 very briefly its geometrical meaning.

3 b) From part a) derive the composite midpoint rule.

4 c) Determine the highest degree of polynomials for which the midpoint
rule is exact

7 d) Use considerations of symmetry and Hermite interpolation to derive
the truncation error formula for the simple midpoint rule.

4 e) Derive from d) the truncation error formula for the composite rule.

4. a) Find the truncation error formula for the forward difference
3 approximation to the first derivative.

5 b) Find a more precise form of the truncation error formula, suitable for
accelerating convergence.

7 c) On the basis of your answer in b) explain exactly how to accelerate the
convergence. Justify your answer.