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Math 128A
First Midterm Exam

October 13, 1998

SHOW YOUR WORK COMPLETELY AND NEATLY. Total points = 60.

- 3 1. a) State the simple trapezoid rule for numerical integration, and indicate very briefly its geometrical meaning.
- 3 b) From part a) derive the composite trapezoid rule.
- 3 c) Determine the highest degree of polynomials for which the trapezoid rule is exact
- 7 d) Derive the truncation error formula for the simple trapezoid rule.
- 4 e) Derive from d) the truncation error formula for the composite rule.
- 5 f) Give a precise discussion of the effect of round-off error when the composite trapezoid rule is used for numerical integration.
- 3 2. a) For distinct centers c_j and values f_j define what is meant by $f[c_0, \dots, c_n]$.
- 11 b) Give a careful proof of the formula
$$f[c_0, \dots, c_n] = (f[c_1, \dots, c_n] - f[c_0, \dots, c_{n-1}])/(c_n - c_0).$$
- 11 3. Find the Newton form of the polynomial which Hermite interpolates the function $f(x) = |x|$ at the points $1, 1, -1, -1$.
- 10 4. Give a brief but precise explanation (including geometry) of how one obtains the formula for Newton's method for finding the zeros of a function. (Include a statement of the formula.)