Math 140 Midterm
March 23, 2000 Prof. Wu

1. (20%) Let $\alpha : [a,b] \to \mathbb{R}^3$ be a regular curve. Show that there exists a function $h : [c,d] \to [a,b]$ so that the curve $\beta = \alpha \circ h : [c,d] \to \mathbb{R}^3$ is a unit speed curve.

2. (15%) If γ is a unit speed curve in $\mathbb{R}^3$, compute $\langle \gamma' \times \gamma'', \gamma''' \rangle$ in terms of curvature and torsion.

3. (15%) Prove that a unit speed curve with zero torsion is a plane curve.

4. (15%) Let γ be a unit speed curve on the sphere of radius r centered at a point $\vec{m}$. Prove that the normal curvature of γ is constant. Explain clearly.

5. (15%) Let $\vec{x} : \mathbb{R}^2 \to \mathbb{R}^3$ be the map $\vec{x}(u,v) = (u+v, u-v, uv)$. Show that $\vec{x}$ is a coordinate patch, and describe the surface $\vec{x}(\mathbb{R}^2)$.

6. (20%) Let $\alpha : \mathbb{R} \to \mathbb{R}^3$ be a simple, unit speed curve and let $\alpha(s) = (\alpha_1(s), \alpha_2(s))$. Let M be the surface $\{ (\alpha_1(s), \alpha_2(s), t) : s, t \in \mathbb{R} \}$. Describe all the geodesics in M, and supply detailed explanations.