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Math 16B, Final Exam, Fall 1996
R. Hartshorne

Part I. Shorter questions. Show work and put answers in boxes. 3 points each. No partial credit. No credit without work shown.

1. Find $\dfrac{\partial}{\partial x}\left(\dfrac{\sin x + \cos y}{\sin x - \cos y}\right)$ and simplify.

2. Find $\displaystyle\int x^2 e^{-x} dx.$

3. Find $\displaystyle\int_e^{e^2} \frac{dx}{x \ln x}$

4. Find $\displaystyle\int_0^{\infty} x e^{-x^2} dx.$

1

5. Use the fact that a circle of radius r has area $A = \pi r^2$ to find the area of the ellipse $9x^2 + 25y^2 = 225$.

6. If $y' = 3t + ty$ and $y(0) = 5$, find $y = f(t)$.

7. If $y' = 3t + t^2$ and $y(0) = 5$, find $y = f(t)$.

8. Find the rational number, **in lowest terms**, whose decimal expansion is .027027**027**...

9. Find the sum of the infinite series $2 + \frac{4}{5} + \frac{8}{25} + \frac{16}{125} + \frac{32}{625} + \ldots$

10. Use a Taylor series to approximate the definite integral $\int_0^{0.1} e^{x^2}\,dx$ to ten decimal places.

Part II. Longer questions. 10 points each. Show your work and put answers in boxes. No credit without work.

1. Let $f(x,y) = 2x^2 - x^4 - y^2$.
   (a) Find all points at which $f(x,y)$ has a potential relative maximum or minimum.

   (b) Use the second derivative test at each of the points found in part (a) above, to determine whether the function has a relative maximum, a relative minimum. neither of these, or no conclusion from the test.

   a)

   b)

2. Integrate
   (a) $\int \sin^3 x\,dx$. Hint: Use the identity $\sin^2 x + \cos^2 x = 1$.

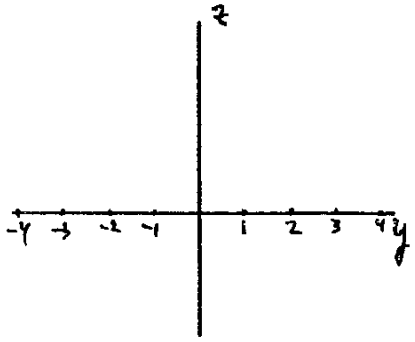
   a)

   (b) $\int x\sec^2 x\,dx$.

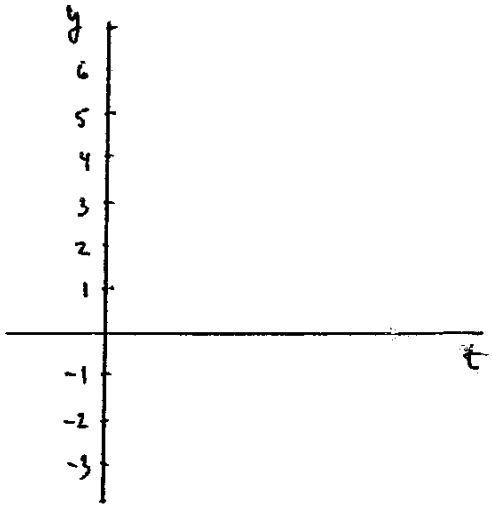
   b)

3. Consider the differential equation $y' = y^2 - 3y - 4$.

   (a) Draw the graph of $z = y^2 - 3y - 4$ in the yz-plane.



   (b) Sketch solutions of the differential equations in the ty-plane, showing constant solutions and the solutions with initial conditions $y(0) = 0$ and $y(0) = 3$. Indicate where the solutions are concave up, or concave down, and mark any inflection points.

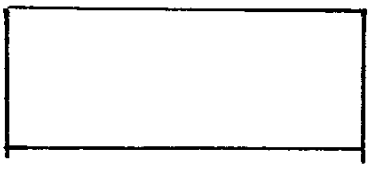


4. Find the first three nonzero terms of the Taylor series for $f(x) = \tan x$ around $x = 0$. (Be sure to write your answer in simplest form.)

5. Given the Taylor series $\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots$

(a) Find the first five terms of the Taylor series for $\ln(1 + 2x)$.

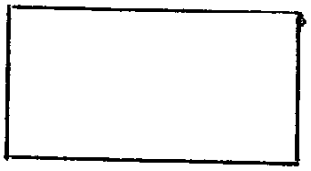
(b) Find the function (in simplest form) whose Taylor series is $2 + 3x + 4x^2 + 5x^3 + 6x^4 + \ldots$. **Hint:** compare to the derivative of the series for $1/(1 - x)$.



6. The XYZ musical instrument company plans to make x xylophones and y yellow synthesizers. Because of restrictions on the time of the expert technicians and the raw materials needed, x and y must satisfy the equation $4x^2 + 25y^2 = 50,000$. The company makes of profit of \$20 for each xylophone and \$100 for each yellow synthesizer.

   (a) Find the production levels x and y which will maximize profits, and

   (b) Find the resulting profit to the company.



7. Suppose that your parents set up a fund for your college education. They deposit \$40,000 into a bank account on January 1st of your first year. (We will assume for simplicty that you start school in January.) This account earns interest, compounded continuously, at a rate of 6% per year. You have to make continuous withdrawals at the rate of \$1,000 per month to pay for your tuition, room and board, etc. (We will also assume that your expenses are spread out evenly throughout the year.)

(a) Write a differential equation for y = the amount of money in the account at time t in years.

(b) Solve the equation to find the function $y = f(t)$.

(c) Now answer this question: Will you be able to complete four years of college with this fund, or will you have to get a job to supplement your income? If the money will run out before the end of four years, in which month of which year will that happen?