Mathematics 16B
Sarason

December 14, 2005

FINAL EXAMINATION

Name (Printed): ____________________

Signature: ____________________

SID Number: ____________________

GSI (check one):
- ☐ Matt Gagliardi
- ☐ Jon Harel
- ☐ James Kelley

Section Number or Time: ____________________

1	
2	
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4	
5	
6	
7	
8	
9	
10	
TOTAL	
GRADE POINTS	

Put your name on every page.

Closed book except for two crib sheets. No Calculators.

SHOW YOUR WORK. Cross out anything you have written that you do not wish the grader to consider.

The points for each question are in parentheses. Perfect score = 150.

Name ______________________________

1. (10) What values of a and b minimize the function

$$E(a,b) = (b - a + 3)^2 + b^2 + (b + a - 2)^2?$$

2. (15) Let the continuous random variable X have density function $f(x) = \frac{3}{2}\sqrt{x}$, $0 \leq x \leq 1$. Compute the expected value $E(X)$ and the variance $\mathrm{Var}(X)$.

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3. (20) The number of pairs of shoes the Phlim Zee Shoe Company can manufacture per week with the utilization of x units of labor and y units of capital is given by the production function $f(x, y) = 40x^{3/4}y^{1/4}$. Each unit of labor costs the company \$200 and each unit of capital costs it \$1,000. To manufacture 1,600 pairs of shoes per week at minimum cost, how many units of labor and how many units of capital should the company utilize? What is the corresponding ratio of labor costs to capital costs?

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4. (15) Perform the integrations.

(a) $\int_0^\pi x \sin x \, dx$ (b) $\int_0^3 x\sqrt{1+x}dx$

(c) $\iint_R (x^2 - \sqrt{y})dxdy$, where R is the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$.

Name ______________________________

5. (20) For the differential equation $2y' = (e - y)^3 e^t$.

   (a) Find the general solution.

   (b) Find the solution satisfying $y(0) = e$.

   (c) Find the solution satisfying $y(0) = e - 1$.

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6. (20) The Dinky University Molecular Biology Scholarship Fund starts the year with assets of \$1,000,000, invested so as to earn interest at the rate of 10% per year, compounded continuously. The fund receives donations at the rate of \$200,000 per year. Assume that donations are received continuously at the preceding rate, and that expenditures (grants plus expenses) occur continuously at the rate of A dollars per year.

   (a) Set up a differential equation satisfied by the DUMB Fund's assets $P(t)$ at time t (measured in years).

   (b) Find the general solution of the differential equation.

   (c) Find the solution satisfying the initial condition $P(0) = 1,000,000$.

   (d) What value must A have in order for the DUMB Fund's assets to be \$1,000,000 at the end of the year? Be clear about how you arrive at your answer.

Name ______________________________

7. (15)

(a) Find the third Taylor polynomial $p_3(x)$ at $x = 0$ for the function $f(x) = \ln(1+x)$.

(b) Use the result from (a) to estimate ln 1.1.

(c) Use the estimate of the remainder $R_3(.1)$ to get an upper bound for the error in the estimate made in (b).

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8. (15) Let the random variable X denote the outcome of rolling a fair pair of dice. The possible values of X are $2, 3, \ldots, 12$, and their probabilities are given in the table below.

Outcome	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

(a) Compute the probabilities $\Pr(X < 7)$, $\Pr(6 \leq x \leq 8)$, $\Pr(X \text{ is odd})$.

(b) Suppose someone gives you 2-to-1 odds that you will not roll 6, 7 or 8. Thus, if the bet is for \$1, you will win \$2 if you roll 6, 7 or 8, otherwise you will lose \$1. What is the expected value of your winnings or losses, as the case may be?

Name ______________________________

9. (10) Suppose the possible values of a discrete random variable X range over the non-negative integers, with $\Pr(X = n) = 3^n/4^{n+1}$ $(n = 0, 1, 2, \ldots)$. Compute $\Pr(X \geq 3)$.

10. (10) Find the Taylor series at $x = 0$ for the function

$$f(x) = \frac{x}{(1-x)^2}.$$