Mathematics 16B
Sarason

May 16, 2006

FINAL EXAMINATION

Name (Printed): ______________________________

Signature: ______________________________

SID Number: ______________________________

GSI (check one):
- ☐ Tom Dorsey
- ☐ Zak Mesyan
- ☐ David Penneys
- ☐ Arun Sharma

Section Number or Time: ______________________________

1	
2	
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5	
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7	
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9	
TOTAL	
GRADE POINTS	

Put your name on every page.

Closed book except for two crib sheets. No Calculators.

SHOW YOUR WORK. Cross out anything you have written that you do not wish the grader to consider. If you continue the answer to a question on the back of the page, put a note to that effect on the front of the page. Make sure the grader can easily spot your final answer(s) to each question, for example by boxing or circling answers where appropriate.

The points for each problem are in parentheses.
Perfect score = 140.

Table of Natural Logarithms (to Four Decimals)

x	$\ln x$
2	.6931
3	1.0986
4	1.3863
5	1.6094
6	1.7918
7	1.9459
8	2.0794
9	2.1972
10	2.3026

Name ______________________________

1. (15) Evaluate the integrals:

(a) $I_1 = \iint_R e^{x+y} dx\, dy$, where R is the triangle with vertices $(0,0)$, $(1,0)$, $(1,1)$.

(b) $I_2 = \int_0^\infty x e^{-x^2} dx$ (c) $I_3 = \int_0^{\pi^2} \sin\sqrt{x}\, dx$

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2. (15) Let

$$E(a,b) = \iint_R [(x-a)^2 + (y-b)^2]\,dx\,dy,$$

where R is the square with vertices $(0,0)$, $(1,0)$, $(0,1)$, $(1,1)$. For which (a,b) is $E(a,b)$ a minimum?

Name ______________________________

3. (20) The Pauvre Suceur Gambling Accessories Manufacturing Company has a contract to produce 960,000 decks of cards. For the plant where the cards are made, the production function $f(x, y) = 12,000x^{2/3}y^{1/3}$ gives the number of decks that can be produced with the utilization of x units of labor and y units of capital. Each unit of labor costs \$1,000 and each unit of capital costs \$4,000.

   (a) Write down the function $g(x, y)$ giving the cost to the company when it utilizes x units of labor and y units of capital.

   (b) Determine the values of x and y that minimize the cost of producing 960,000 decks of cards. Use Lagrange's method and take care not to confuse the objective and constraint functions. (You will lose points if you do confuse them.)

   (c) Compare labor costs with capital costs for the minimizing values of x and y.

Name ______________________________

4. (20) (a) Find the general solution of the differential equation

$$2yy' = -(y^2 - 1)^2.$$

(b) Find the solution satisfying the initial condition $y(1) = -2$.

(c) Find the solution satisfying the initial condition $y(1) = -1$.

Name ______________________________

5. (20) Bianca Confucion takes out a \$500,000 mortgage to buy a hovel near the Berkeley campus. The yearly interest rate is 5%, compounded continuously, and yearly payments are \$35,000, applied continuously.

   (a) Set up a differential equation satisfied by the unpaid amount $P(t)$ of the mortgage at time t (with t measured in years).

   (b) Find the general solution of the differential equation.

   (c) Find the solution satisfying the initial condition $P(0) = 500,000$.

   (d) Determine how long it will take Bianca to repay the loan in full. (You will need to use the logarithm table on the cover sheet.)

Name ______________________________

6. (15) (a) Find the third Taylor polynomial $p_3(x)$ at $x = 1$ for the function $f(x) = \sqrt{x}$.

(b) Use the result from (a) to estimate $\sqrt{1.2}$. Express your answer in decimal form.

(c) Use the remainder estimate to get a bound on the error in the approximation obtained in (b). Again, express your answer in decimal form.

Name ______________________________

7. (15) For a continuous random variable X with probability density function $f(x) = \sin 2x$, $0 \leq x \leq \frac{\pi}{2}$, compute the expected value $E(X)$ and the variance $\mathrm{Var}(X)$.

Name ______________________________

8. (10) Suppose the possible values of the discrete random variable X range over the nonnegative integers, and the associated probabilities are given by $p_n = Pr(X = n) = 6^n/7^{n+1}$ $(n = 0, 1, 2, \dots)$. Compute $Pr(X$ is even$)$.

9. (10) (a) Derive the formula

$$\int_a^b x^2 e^{-x^2/2} dx = \int_a^b e^{-x^2/2} dx + ae^{-a^2/2} - be^{-b^2/2}.$$

(b) Let X be a standard normal random variable, i.e., a continuous random variable whose density function is the function $\psi(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$, $-\infty < x < \infty$. Use the result from (a) to show that $\mathrm{Var}(X) = 1$.