Math 16b, **Final Exam**

Wednesday, December 12, 12:30–3:30pm

This exam is closed book. You may not use any books, notes, or calculators. Please write your answers in a blue note book. Write your name, the name of your TA and your section time on the cover. There are ten problems, each worth ten points. Answers without justification will not receive credit.

(1) The amount of space required by a start-up company is

$$f(x,y) \quad = \quad 1000 \cdot \sqrt{6x^2 + y^2},$$

where x and y are the numbers of units of labor and capital utilized. Labor costs \$ 200 per unit and capital costs \$ 100 per unit, and the firm has \$ 10,000 to spend. Determine the amounts of labor and capital that should be utilized in order to minimize the amount of space required.

(2) Let T be the triangle in the plane with vertices $(0,0)$, $(1,0)$ and $(1,1)$. Calculate the double integral

$$\iint_T e^{2x-3y}\, dxdy.$$

(3) Compute the following two indefinite integrals:

(a)

$$\int \frac{\cos x}{(7+\sin x)^4}\, dx$$

(b)

$$\int x^2 \sin x\, dx.$$

(4) A rocket takes off. The velocity readings recorded every second during the first five seconds of the rocket's flight were 0, 30, 70, 130, 170 and 200 feet per seconds. Use the **trapezoidal rule** to estimate the distance the rocket traveled during the first five seconds of its flight.

(5) Solve the following differential equation with given initial condition:

$$y' \; = \; t \cdot e^{t+y}, \quad y(0) = 0.$$

(6) Show that the following infinite series converges and determine its sum:

$$\sum_{k=2}^{\infty} \frac{3^k + 5^{k+2}}{2^{3k}}.$$

(7) Using sum notation, give the Taylor series at $x = 0$ of the function

$$f(x) \quad = \quad x^5 \cdot (e^{-x} - 1).$$

(8) Determine whether the following series are convergent or divergent:

(a)

$$\sum_{k=3}^{\infty} \frac{1}{k \cdot (\ln k)^3}$$

(b)

$$\sum_{k=2}^{\infty} \frac{e^{1/k}}{k^2}$$

(9) The time (measured in hours) between two northbound trains at the Berkeley BART station is a random variable with density function

$$f(x) \quad = \quad 5(1-x)^4 \qquad \text{for} \quad 0 \leq x \leq 1.$$

(a) What is the probability that the time between two northbound trains is more than thirty minutes ?

(b) Compute the average time between two northbound trains.

(10) Scores on a school's entrance exam are normally distributed, with mean $\mu = 500$ and standard deviation $\sigma = 100$. If the school wishes to admit only the students in the top 40%, what should be the cutoff grade ? *(Hint: You may use the attached Table)*

ppendix E
Areas under the Standard Normal Curve

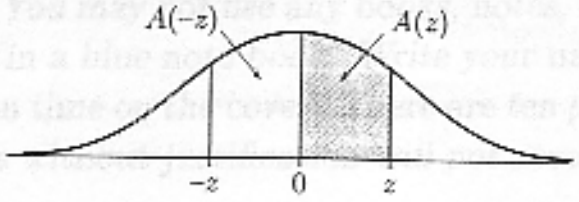

Table 1 **Areas under the Standard Normal Curve**

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0754
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2258	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2518	.2549
0.7	.2580	.2612	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2996	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3820
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998