

Name _____

TA's Name _____

Section _____

Math 16B
R. Hartshorne

Final Exam, December 11, 2003

Do Not Write Here

1	2	3	4	5	6	
7	8	9	10	11	12	
1						
2						
3						
4						
Total						

Part I. Shorter questions. 5 points each.
Show work and put answers in boxes.
No partial credit. All answers must be
in simplest form. No calculators. You
may leave expressions such as π , e , $\sqrt{2}$
in answers.

1. Find $\frac{\partial}{\partial y} \left(\frac{\sin x + \cos y}{\sin x - \cos y} \right)$ and simplify.

2. If $\sin t = -\frac{1}{3}$ and $-\frac{\pi}{2} < t < \frac{\pi}{2}$, find $\tan t$.

3. Find the total area between the curve $y = \frac{1}{x^2 + 4}$
and the x -axis.

4. If $y' = 5t + 3ty$ and $y(0) = 1$, find $y = f(t)$.

1.

2.

3.

4.

5. $\int x^2 \sin x \, dx =$

6. $\int \cos^3 x \, dx =$

7. Use the Taylor series for $\sin x$ to compute $\sin(0.3)$ to 6 decimal places.

8. Find the 5th Taylor polynomial of $y = \tan x$.
Reduce fractions to lowest terms.

9. Use two iterations of the Newton-Raphson algorithm, starting with $x_0 = 2$, to find an approximation for $\sqrt{3}$. Leave your answer as a fraction in lowest terms.

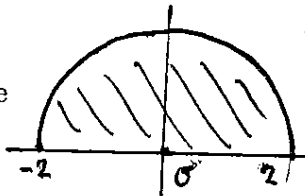
10. Find the sum of the infinite series

$$1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{6} + \frac{1}{6^2} + \frac{1}{6^3} + \cdots + \frac{1}{6^n} + \cdots$$

Leave your answer as a fraction in lowest terms.

11. Find the rational number (as a fraction in lowest terms) whose decimal is $0.135135\overline{135}\dots$

12. Experiment: Pick a point at random in the half disk of radius 2. Let X be the distance from the center O . Find the probability density function for the random variable X .



Part II. Longer problems. 10 points each.
Show your work. Put answers in boxes.

1. Evaluate $\int_0^{\frac{1}{2}} \sqrt{1-x^2} dx$.

- a) Make a trigonometric substitution, and write the new integral with new limits of integration.
b) Evaluate the integral to find the answer.

a.
b.

2. Find the maximum value attained by the function $y = 4 \sin x + 3 \cos x$ on the interval $0 \leq x \leq \pi$.

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3. For each of the following, determine if the infinite series converges or diverges. State which method you use and show your work.

a) $\sum_{k=1}^{\infty} \frac{5}{2^k + 3}$

b) $\sum_{k=1}^{\infty} \frac{5}{2k + 3}$

c) $\sum_{k=1}^{\infty} \frac{5}{k^2 + 3}$

a
b
c

4. a) Find the Taylor series for $f(x) = \frac{1}{1+x}$.

b) Find the Taylor series for $\ln(1+x)$.

c) Find the Taylor series for $\frac{\ln(1+x^2)}{x^2}$.

a
b
c