Department of Mathematics, University of California, Berkeley

Math 16A

Alan Weinstein, Fall 1995

Second Midterm Exam, Monday, November 6, 1995

Instructions. BE SURE TO WRITE YOUR NAME AND YOUR TA'S NAME AND SECTION NUMBER ON YOUR BLUE BOOK. Read the problems very carefully to be sure that you understand the statements. All work should be shown in the blue book; writing should be legible and clear. Indicate the final answers to problems by circling them. [Point values of problems are in square brackets. The total point value is 45, for 15% of your course grade.] **You may keep this exam paper; do not hand it in with your blue book.**

1. [7 points] Find the maximum and minimum values of $f(x) = x^4 - 8x^2$ for x in the interval $[-1, 3]$.

2. [8 points] Sketch the graph of a function $y = f(x)$, given the following information about f. Label the point(s) of inflection and the local extreme point(s). (Your solution should consist of a *single* sketch.)

- $f(x)$ is positive for $x < 2$ and $f(x)$ is negative for $x > 2$.
- $f'(x)$ is positive for $x < 1$ and $f'(x)$ is negative for $x > 1$.
- $f''(x)$ is positive for $x < 0$ and $f''(x)$ is negative for $x > 0$.

3. [8 points] A ship uses $5x^2$ dollars of fuel *per hour* when traveling at a speed of x miles per hour. The other expenses of operating the ship amount to \$2000 per hour. What speed minimizes the cost of a 1000 mile trip? [*Hint*: Use the fact that speed = distance ÷ time.]

4. [9 points]

a.

$$\text{Find } \frac{dy}{dx} \text{ if } y = \frac{4 - x^2}{4 + x^2}.$$

b.

$$\text{Find } f'(x) \text{ if } f(x) = x^4 e^x.$$

c.

$$\text{If } x^2 + y^4 = 2, \text{ find } \frac{dy}{dx} \text{ when } x = 1 \text{ and } y = 1.$$

5. [7 points] Write a complete sentence or two (but not more) expressing clearly and precisely the relation between the **second derivative** of a function f and the **slopes of the tangent lines to the graph of** f.

6. [6 points] The tangent line to the graph of g at $x = 2$ has slope 5. $f(x) = 5x^2$. $g(6) = 4$, $g(2) = 6$, $g'(12) = 8$, and $g'(6) = 14$. Find the equation of the tangent line to the graph of $f(g(x))$ at $x = 2$.