Math 130 F95
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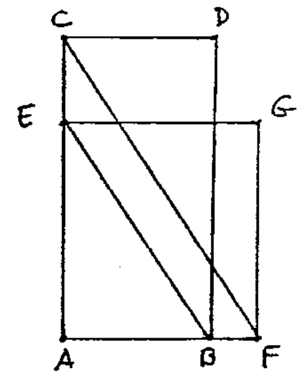
FINAL EXAM

Name ______________

1. Make a ruler and compass construction of three circles, each orthogonal to the other two. Label points and number your steps as usual. Explain briefly why your construction works.

2. Construct a regular pentagon having AB given for one of its sides. Label and number your steps as usual



3. In Euclidean geometry, suppose given a rectangle ABCD, and given a point E on AC so that $AE > \frac{1}{2}AC$. Draw EB, draw CF parallel to EB (this gives F), and construct the rectangle AFEG.

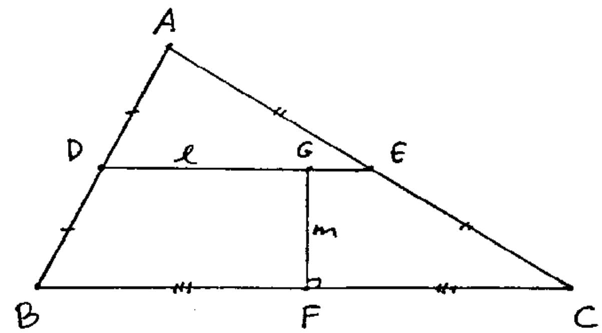
Prove, in the manner of Euclid, that the rectangle ABCD has the same content as the rectangle AFEG. Quote by Book and number any propositions from Euclid you wish to use.

4. In a Hilbert plane, show that the following three properties are equivalent:

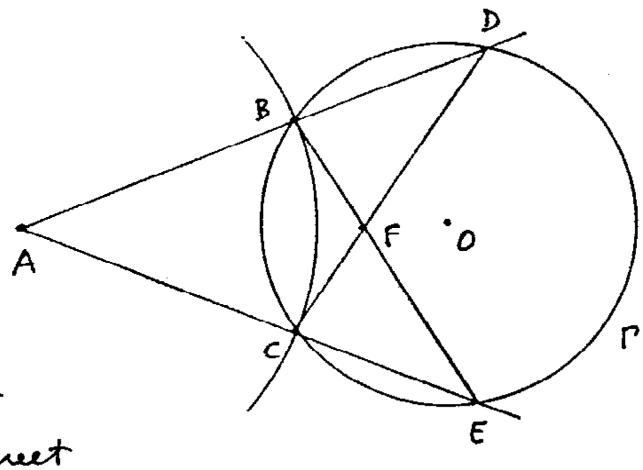
(i) The line-circle intersection property (LC)

(ii) For any point A outside a circle Γ, there exists a line through A tangent to Γ.

(iii) For any two line segments AB, CD, with $AB < CD$, there exists a right triangle with one leg $\cong AB$ and with hypotenuse $\cong CD$.

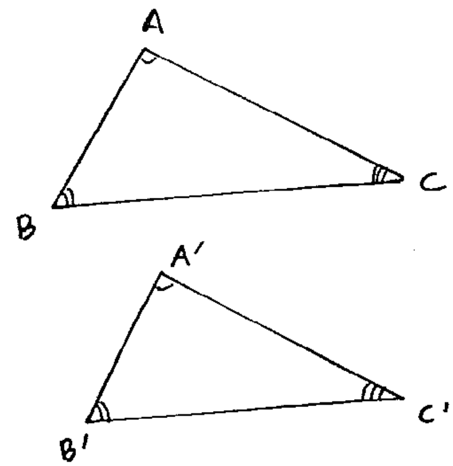


5. In a Hilbert plane, let ABC be any triangle, and let D, E, F be the midpoints of the sides. Let l be the line DE and let m be the line through F, perpendicular to BC. Prove that l meets m at a point G and that l and m make a right angle at G.

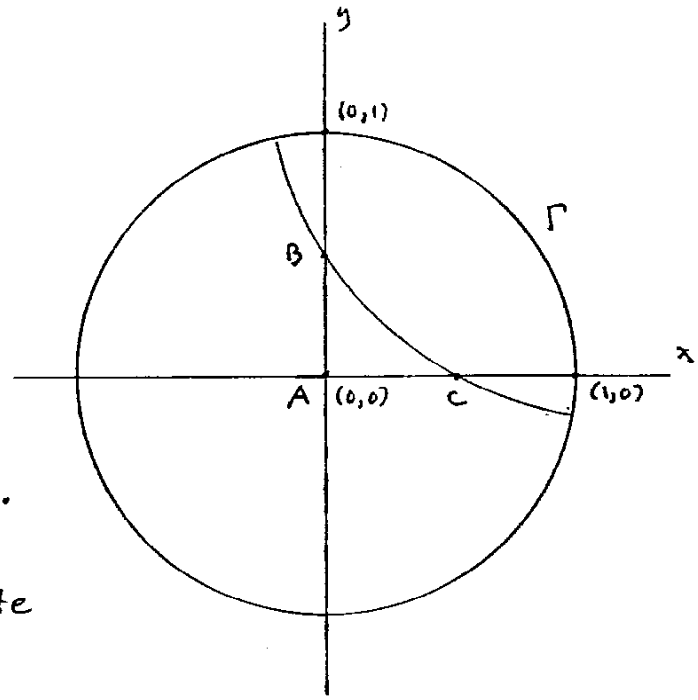


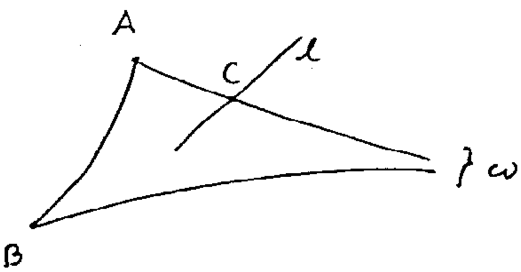
6. In the Euclidean plane over a field F satisfying (**), consider the following construction. Given a circle Γ and a point A outside Γ. Draw a circle with center A, any radius, to meet Γ in B, C. Draw AB, AC to meet Γ in D, E. Draw BE, CD, get F. Prove that F is the circular inverse of A in Γ.

7. In hyperbolic geometry, suppose two triangles ABC and $A'B'C'$ have their three angles respectively congruent: $\angle A \cong \angle A'$, $\angle B \cong \angle B'$, and $\angle C \cong \angle C'$. Prove that the sides are also congruent.

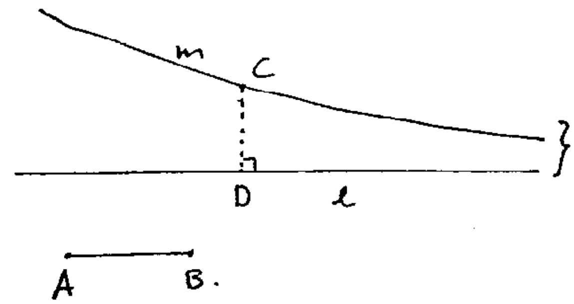


8. In the Euclidean plane over a field F, take Γ to be the unit circle, and consider the Poincaré model of hyperbolic geometry using Γ. Let $A=(0,0)$, $B=(0,\frac{1}{2})$, $C=(\frac{1}{2},0)$. Let δ be the defect of the hyperbolic triangle ABC. Compute $\tan\delta$.





9. In a hyperbolic plane, let $AB\omega$ be a limit triangle, i.e. $A\omega$ and $B\omega$ are two limiting parallel rays. Suppose a line ℓ meets the ray $A\omega$ at a point C. Prove that ℓ either meets the segment AB, or passes through B, or meets the ray $B\omega$. (This is the analogue of Pasch's axiom for limit triangles.)



10. In a hyperbolic plane, suppose given two limiting parallel lines l, m, and given a segment AB. Prove that there is a point $C \in m$, such that the perpendicular CD to l has $CD \cong AB$.