Math 130 Midterm I

F 2002 September 23, 2002 Wee

1. (20%) Let T be the translation of $\mathbb{R}^2$ by $P = (3, -2)$ ($T(Q) = Q + P$ for every $Q \in \mathbb{R}^2$), let ρ be the rotation of $+90°$ around the origin, and let R be the reflection with respect to the line $x - y = 0$. What is the point $(R \circ T \circ \rho)(1, 2)$?

2. (20%) Let R be the reflection with respect to a line $\mathcal{L}$. Suppose L is a line so that $R(L) \parallel L$. Prove that $\mathcal{L} \parallel L$.

3. (30%) Let L be a line in $\mathbb{R}^2$ and let $\mathbb{R}^2 = \mathbb{R}^2_+ \cup L \cup \mathbb{R}^2_-$ be the usual decomposition of $\mathbb{R}^2$ as a union of L and the two half-spaces of L. Suppose $\mathcal{C}$, $\mathcal{D}$ are convex sets in $\mathbb{R}^2$ so that (1) $\mathcal{C}$, $\mathcal{D}$, L are disjoint and (2) $\mathbb{R}^2 = \mathcal{C} \cup L \cup \mathcal{D}$. Prove that $\mathcal{C} = \mathbb{R}^2_+$ or $\mathbb{R}^2_-$.

4. (25%) Let $-\pi \leq \theta \leq \pi$, and let ρ_θ with rotation of (oriented) angle θ around the origin. Prove that $\rho_\theta \circ \rho_{-\theta} = id$, where id = identity map of $\mathbb{R}^2$.

5 (5%) How many hours on average do you spend on this course per week? Of these, how many are devoted to doing the homework?