Math 130 Final H. Wu

Dec 17, 2002

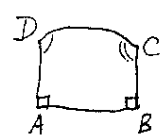
Hillbert Plane

1. In Hilbert plane + (P), (a) prove that any two altitudes of a triangle must meet, and (b) prove that all three altitudes of a triangle are concurrent, i.e. meet at a point. (30 points)

2. In a Hilbert plane, prove AAS. More precisely, suppose two triangles, $\triangle ABC$ and $\triangle A'B'C'$ satisfy $\overline{AB} \cong \overline{A'B'}$, $\angle A \cong \angle A'$ and $\angle C \cong \angle C'$, then $\triangle ABC \cong \triangle A'B'C'$. (30 points)

3. In Hilbert plane + (A), if there exists one triangle with angle sum = 180, then (P) must hold. (20 points)

4. In a Hilbert plane, suppose the top angles $\angle C$ and $\angle D$ of a Saccheri quadrilateral are obtuse, then $\overline{AB} > \overline{DC}$. (20 points)



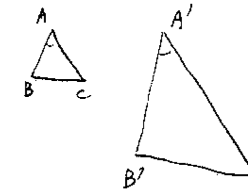
Coordinate Geometry

5. Prove that through a point P not on a line ℓ, there is one and only one line L such that $L \perp \ell$. (30 points)

6. Prove SAS for similar triangles. More precisely, suppose $\triangle ABC$ and $\triangle A'B'C'$ are given such that $\angle A \cong \angle A'$ and

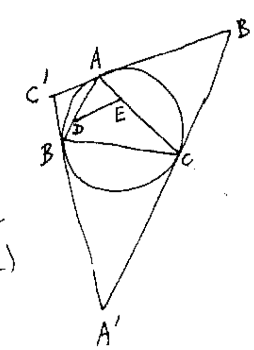
$$\frac{|\overline{AB}|}{|\overline{A'B'}|} = \frac{|\overline{AC}|}{|\overline{A'C'}|}.$$

Then $\triangle ABC \sim \triangle A'B'C'$. (20 points)



Classical Euclidean Geometry

7. In $\triangle ABC$, let $D \in AB$, $E \in AC$. Prove $\overline{DE}$ is an antiparallel to $\overline{BC}$ iff $DE \parallel$ the tangent to the circumcircle of $\triangle ABC$ at A ($C'B'$ in the picture) (30 points).



8. Let the tangents to the circumcircle of $\triangle ABC$ at the vertices meet at A', B', C'. Prove that AA' bisects every antiparallel to $\overline{BC}$. (20 points)