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Spring 2001, Math 114

Second Midterm

21 March, 2001
12:10-1:00 PM

1. (48 points; 12 points each.) For each of the items listed below, either *give an example,* or give a brief reason why *no example exists.* (If you give an example, you do *not* have to prove that it has the property stated.)

(a) A radical extension $L:K$ which is not normal.

(b) A simple group of order 125.

(c) Two distinct intermediate fields in the extension $\mathbb{Q}(\sqrt[4]{2}, i):\mathbb{Q}$ which are isomorphic to each other. (''Distinct'' means ''not equal to each other''.)

(d) Two isomorphic groups $G \cong H$ such that G is simple but H is not.

2. (30 points) Suppose $L:K$ is a separable normal extension of degree p^2q^2, where p and q are distinct primes.

(a) Show that there exists an intermediate field M having degree p^2 over K. (Suggestion: Use the Fundamental Theorem of Galois Theory, and some group theory you have learned.)

(b) Name an integer $m \neq 1, p^2, q^2, p^2q^2$, such that $L:K$ must have an intermediate field M of degree m over K, and prove your assertion.

3. (22 points) Suppose $L:K$ is a finite normal algebraic extension, and M an intermediate field. Show that $L:M$ is also normal. (This is a fact that Stewart uses, referring the reader to a previous result but not saying how it follows from that result. You should give an explicit argument. We discussed it in class more than once.)