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Spring 2001, Math 114  
First Midterm

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12:10-1:00 PM

1. (30 points, 10 points each.) Complete each of the following definitions. (Do not give examples or other additional facts about the concepts defined.)

(a) If  $L:K$  is a field extension, and  $\alpha$  is an element of  $L$ , then  $\alpha$  is said to be *algebraic* over  $K$  if

(b) If  $K$  is a field and  $X$  a subset of  $K$ , then the subfield of  $K$  generated by  $X$  is defined to be

(c) If  $K$  is a subfield of each of the fields  $M$  and  $L$ , then a monomorphism  $\varphi: M \rightarrow L$  is said to be a *K-monomorphism* if

2. (50 points; 10 points each.) For each of the items listed below, either *give an example*, or give a brief reason why *no example exists*. (If you give an example, you do *not* have to prove that it has the property stated.)

(a) A field extension  $L:K$ , and a polynomial  $f \in K[t]$  which is irreducible over  $K$  but reducible over  $L$ .

(b) A field extension  $L:K$ , and a polynomial  $f \in K[t]$  which is reducible over  $K$  but irreducible over  $L$ .

(c) A finite field extension  $L:K$  such that, regarding  $K$  as an intermediate field, we have  $K^{*\dagger} = K$ .

(d) A finite field extension  $L:K$  such that, regarding  $K$  as an intermediate field, we have  $K^{*\dagger} \neq K$ .

(e) A finite field extension  $L:K$  and three distinct automorphisms  $\alpha, \beta, \gamma \in \Gamma(L:K)$  such that for all  $x \in L$ ,  $\alpha(x) + \beta(x) + \gamma(x) = 0$ .

3. (20 points) (a) (10 points) Suppose  $L:K$  is a field extension, and  $H$  a subgroup of  $\Gamma(L:K)$ . Recall that  $H^\dagger$  means  $\{x \in L \mid (\forall \alpha \in H) \alpha(x) = x\}$ . In Lemma 7.2 Stewart shows that  $H^\dagger$  is a subfield of  $L$  containing  $K$ . Prove the two parts of that statement saying that  $H^\dagger$  is closed in  $L$  under addition and under multiplication.

Since I am asking you for the proof of a result in Stewart, you may not, of course, cite that result, or any result proved after it, in your proof.

(b) (10 points) Prove that  $H^\dagger$  is also closed under multiplicative inverses (i.e., that if  $x \in H^\dagger$  and  $x \neq 0$  then  $x^{-1} \in H^\dagger$ ). Stewart is not explicit about this point. You should supply the details, though he omits them. (Suggestion: Use the equation by which  $x^{-1}$  is defined. You may assume without argument that every automorphism of a field sends 1 to 1, but no other facts about automorphisms other than the definition.)