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Fall 2001, Math 113, Sec. 3
First Midterm

28 Sept., 2001
11:10-12:00

1. (24 points, 8 points apiece) Find the following.

(a) The remainder r when $29 \cdot 53 - 10^3$ is written in the form $b \cdot 9 + r$ with $0 \leq r \leq 9$.

(b) $(1,2,3,4)(2,4,5)^{-1}$, expressed as a product of disjoint cycles in S_6.

(c) The order of the cyclic subgroup $\langle (1,2)(3,4)(5,6,7) \rangle$ of S_{10}.

2. (36 points; 9 points each.) For each of the items listed below, either *give an example,* or give a brief reason why *no example exists.* (If you give an example, you do *not* have to prove that it has the property stated.)

(a) An integer x such that $x \equiv 27 \pmod{100}$ and $x \equiv 59 \pmod{101}$. (You need not write it out explicitly; you may instead give an arithmetic expression whose value must have that property.)

(b) An element of order 10 in S_9.

(c) An element of order 2 in a finite group of odd order.

(d) An isomorphism ψ between $\mathbf{Z}_3$ and a subgroup of S_3.

3. (20 points) Define what is meant by a *group* $(G, \cdot)$.

(You need not use the exact wording of the definition in the text as modified in my corrections; but for full credit your answer must clearly express all the conditions in that definition, and should avoid the type of poor wording that my corrections changed.)

4. (20 points) Let G be a group. Show that for all $a, b, c \in G$ the equation $axb = c$ has a unique solution in G. (This means showing both that an element x exists which satisfies the equation, and that there is only one such element.)