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Spring 1999, Math 113, Section 2

26 February, 1999
1:10-2:00 PM

First Midterm

1. (35 points, 7 points apiece) Find the following.

(a) $[83]_8 \cdot [55]_8 + ([40]_8)^3$, expressed in the form $[r]_8$ with $0 \leq r < 8$.

(b) The multiplicative inverse of $[9]_{20}$ in $\mathbf{Z}_{20}$.

(c) $(1,2,3,4)^2$, expressed as a product of disjoint cycles in S_4.

(d) The equivalence class $[-2]$ of -2 under the equivalence relation $\sim$ on $\mathbf{R}$ induced by the function $f(x) = x^2$. (List or describe the elements of $[-2]$ within set brackets { }.)

(c) The cyclic subgroup of S_8 generated by the element $(1,3,5)(2,4,6)$. (List the elements within set brackets { }.)

2. (31 points) Suppose I is a subset of $\mathbf{Z}$ which contains at least one positive integer and is closed under addition and subtraction. Prove that there is some positive integer b such that $I = b\mathbf{Z}$ (where $b\mathbf{Z}$ means $\{bn \mid n \in \mathbf{Z}\}$).

(The above is a part of the proof of a result from the reading. In proving it, you may assume the Well-Ordering Principle and the Division Algorithm, but not later results about the integers.)

3. (34 points) In each part below, either give an example, and indicate very briefly why it satisfies the given condition, or show that no such example can exist. (Recall that a group G is called *abelian* or *commutative* if for all $x, y \in G$ one has $xy = yx$.)

(a) (9 points) A nonabelian group G having an abelian subgroup $K \subseteq G$.

(b) (9 points) An abelian group G having a nonabelian subgroup $K \subseteq G$.

(c) (8 points) A binary operation $*$ on a set S, and a subset $T \subseteq S$ which is not closed under $*$.

(d) (8 points) An element of order 15 in $S_3 \times \mathbf{Z}_{10}$.