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Spring 1997, Math 113, Lecture 3 — 21 February, 1997

First Midterm Exam — 1:10-2:00 PM

1. (30 points; 6 points each) Compute each of the following.

(a) The order of the element $32 \in \mathbb{Z}_{100}$.

(b) The value of i^{35} in $\mathbb{C}$.

(c) The inverse of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 6 & 4 & 3 & 2 & 5 \end{pmatrix}$ in S_6.

(d) The expression for $(1,2,6)(6,4,5)(1,4)$ as a product of disjoint cycles in S_6.

(e) The orbit of 7 under the permutation σ of the set of elements of $\mathbb{Z}_8$ given by $\sigma(n) = n+4$ (addition in $\mathbb{Z}_8$). (Note: σ is not itself a member of $\mathbb{Z}_8$.)

2. (30 points) Define what is meant by a ***group***. You may assume the concept of a "binary operation on a set" has been defined, so that you should not repeat that definition. However, do not assume the names of any conditions on such a binary operation have been defined; thus, the meaning of any such condition that you use should be stated explicitly.

3. Let G be a finite group. Below, we will consider elements of order 9 in G. In proving results asked for, you may call on any result proved in the readings if you can state it accurately. You don't need to quote such a result word-for-word, but you should state its content correctly.

(a) (17 points) Show that if a is an element of order 9 in G, then exactly 6 elements of the cyclic subgroup $\langle a \rangle$ have order 9.

(b) (13 points) Show that no two distinct subgroups of G of order 9 have any element of order 9 in common.

(c) (10 points) Let n be the number of *cyclic subgroups* of order 9 in G, and let m be the number of *elements* of order 9 in G. State and justify an equation by which one can compute m from n and vice versa. You may assume the results that you were asked to prove in parts (a) and (b) above, whether or not you succeeded in proving them.