MEAN 144/200
SD 36/200

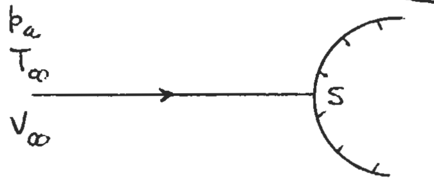
Full credit for correct final answer with coherent argument - otherwise partial credit as shown

ME106 Fluid mechanics
2nd Test, S04

NAME SOLUTIONS

MEAN 52
SD 14

1. (65) Air at temperature $T_\infty = 293$ K and pressure $p_a = 101$ kPa flows at speed $V_\infty = 500$ m/s towards the stagnation point S. The specific heat ratio $\gamma = 1.4$, and the gas constant $R = 287$ J/kg·K. Find: (A SHOCK OCCURS ON THE STAGNATION SL.)



(a) the stagnation temperature T_o;

(20) $\frac{V^2}{2c_p} + T = T_o$ defines stagnation temp. T_o; evaluate LHS at ∞

$c_p = \frac{\gamma}{\gamma-1}R = 1004.5 \Rightarrow T_o = \frac{(500)^2}{2009} + 293 = 417\text{ K} \Rightarrow \boxed{T_o = 417\text{ K}}$

Stagnation temp.

(b) the stagnation pressure p_o at infinity; and

(20) $$\frac{p}{p_o} = \left(\frac{T}{T_o}\right)^{\frac{\gamma}{\gamma-1}}$$ for given p, T this defines stagnation pressure i.e. pressure in the stagnation chamber used to generate the flow p_∞, T_∞. Solve for $p_o = p_\infty\left(\frac{T_o}{T_\infty}\right)^{\gamma/(\gamma-1)} = 101 \times \left(\frac{417}{293}\right)^{\frac{1.4}{0.4}}$, $\boxed{p_o = 347\text{ kPa}}$

stagnation pressure

(c) the pressure p_s at the stagnation point.

(25) From BE $V^2 = 2c_pT_o\left(1 - \left(\frac{p}{p_o}\right)^{1-\frac{1}{\gamma}}\right)$, $p_o = p\Big/\left(1 - \frac{V^2}{2c_pT_o}\right)^{\frac{\gamma}{\gamma-1}}$, gives the stagnation pressure at a point with pressure p and speed V.

-10 Correct eq wrongly used
-5 Minor slip / So with correct approach
-5 Correct eq, wrongly copied.

$$\boxed{\frac{p_{o2}}{p_{o1}} = \frac{p_2}{p_1}\left(\frac{1 - V_1^2/2c_pT_o}{1 - V_2^2/2c_pT_o}\right)^{\frac{\gamma}{\gamma-1}}}$$

UNSHOCKED → 500 m/s ① | ② → SHOCKED

SHOCK STATIONARY

(T_o is same on both sides.)

Here $T_1 = 293\text{ K} \Rightarrow c_1 = \sqrt{\gamma RT} = 343$, $V_1 = 500$ m/s $\Rightarrow$ $\underline{M_1 = 1.46}$

$$\frac{V_1}{V_2} = \frac{(\gamma+1)M_1^2}{(\gamma-1)M_1^2 + 2} = 1.79 \Rightarrow \underline{V_2 = 280\text{ m/s}}$$

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1) = \underline{2.320} \Rightarrow \frac{p_{o2}}{p_{o1}} = 2.32 \times \left(\frac{1 - 500^2/2009 \times 417}{1 - 280^2/2009 \times 417}\right)^{3.5}$$

PLEASE PRINT YOUR NAME ON THIS PAGE

SP042–1

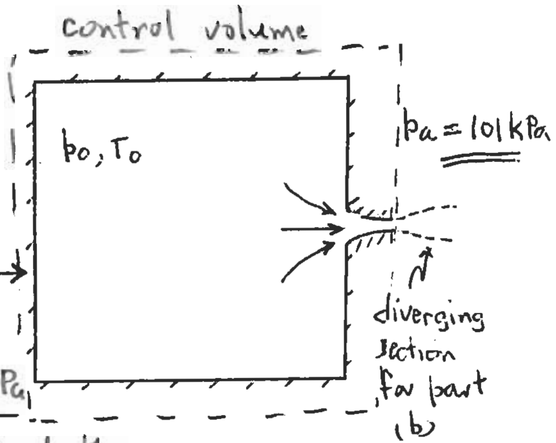
$= 0.946 \Rightarrow \boxed{p_{o2} = p_S = .946 \times 347 = 328\text{ kPa}}$

Stagnation pressure FALLS across shock

NO CREDIT FOR RESULTS OBTAINED BY USE OF INAPPROPRIATE EQ

* DO NOT DO PART (C) *

IN (A) NO CREDIT UNLESS THEY TEST FOR CHOKED FLOW



MEAN 40
SD 19

2. (65) The large tank contains compressed air at pressure $p_o = 400$ kPa and temperature $T_o = 293$ K. (Relevant properties for air are given in problem 1.) Find the horizontal component of force needed to hold the tank stationary if the air leaves to atmospheric pressure under the following conditions:

(a) through a converging nozzle with exit diameter 10 mm; and

CHECK (10)

Sonic pressure $p_* = \left(\frac{2}{\gamma+1}\right)^{\gamma/\gamma-1} p_o = \underline{0.528\, p_o} = 211$ kPa

Because $p_a = 101$ kPa $< p_*$, the flow is choked and the

CONCLUSION (+10)

(35) momentum flow out of the C.V. shown is $\rho_* V_*^2 A$, A is exit area.

RESULT (+5)

Momentum balance $\boxed{F = \rho_* V_*^2 A_* + (p_* - p_a) A_* = 23.2 + 8.6 = 31.8\text{ N}}$

(+5)

$\rho_* = \frac{p_o}{RT_o}\left(\frac{2}{\gamma+1}\right)^{1/\gamma-1} = 0.634 \frac{p_o}{RT_o} = 3.02$ kg/m³, $V_* = \sqrt{\frac{2}{\gamma+1}}\sqrt{\gamma R T_o} = 0.912\sqrt{\gamma R T_o} = 313$ m/s

(+5)

$A_* = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times (10^{-2})^2 = 7.85 \times 10^{-5}$ m²

(b) through a converging–diverging nozzle chosen so the exit pressure is atmospheric.

Because the flow is choked in (a), now the momentum flow is $\rho_* V_* V_e A$

where $V_e^2 = 2c_p T_o \left(1 - \left(\frac{p_a}{p_o}\right)^{1-\frac{1}{\gamma}}\right) \Rightarrow V_e^2 = 2009 \times 293 \times \left(1 - \left(\frac{101}{400}\right)^{1-\frac{1}{1.4}}\right)$

$V_e = 438$ m/s

(30)

& $F = \rho_* V_* V_e A_* = 3.02 \times 313 \times 438 \times 7.85 \times 10^{-5}$

$\Rightarrow \boxed{F = 32.5\text{ N}}$

$\dot{m}$ +15

V_e +15

* DO NOT DO PART (c) *

(c) For part (b), find the exit area of the nozzle.

(from $(\rho A V)^2 = (\rho_* A_* V_*)^2$, Bernoulli eq. & $\rho/\rho_* = (p/p_*)^{1/\gamma}$)

NOT GRADED

$$\left(\frac{A_*}{A_e}\right)^2 = \frac{2}{\gamma-1}\left(\frac{1+\gamma}{2}\right)^{\frac{\gamma+1}{\gamma-1}}\left[\left(\frac{p_e}{p_o}\right)^{2/\gamma} - \left(\frac{p_e}{p_o}\right)^{1+\frac{1}{\gamma}}\right]$$

$$= \frac{2}{.4}\left(\frac{2.4}{2}\right)^{\frac{2.4}{.4}}\left[\left(\frac{101}{400}\right)^{2/1.4} - \left(\frac{101}{400}\right)^{1+\frac{1}{1.4}}\right]$$

$$= 14.93 \times [0.1399 - 0.09447] = 0.679$$

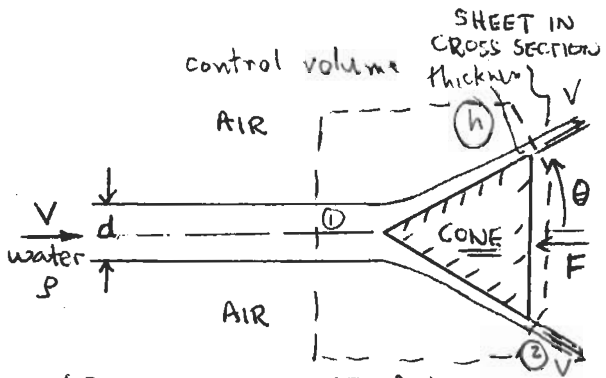
$$\Rightarrow A_e = A_* / \sqrt{.679} = 7.85 \times 10^{-5}\text{ m}^2 / .824 = 9.52 \times 10^{-5}\text{ m}^2.$$

$\Rightarrow \boxed{A_e = 9.52 \times 10^{-5}\text{ m}^2}$

($d = 11$ mm)

MEAN 52
SD 18

3. (70) A jet of water flows over the fixed cone and leaves as a conical sheet. The flow is incompressible and inviscid, and gravity is negligible. Using mass and momentum balances, and Bernoulli's equation, find the force F needed to hold the cone stationary in terms of the diameter d, density ρ, jet speed V and cone angle θ.



(CROSS-SECTION OF AN AXISYMMETRIC FLOW)

(20) By Bernoulli's equ. along the surface streamline, the speed at the exit of the CV is also V. ⇕ These two points (2) (If unif. V assumed without proof −5)

(25) Mass balance requires mass flow rate in (1) = $\dot{m}$ = mass flow across ring (2)

(25) Balance of horizontal momentum   Correct momentum flows +25

Net flow of horiz. mom. out $= \dot{m}V\cos\theta - \dot{m}V$
= resultant external force on all matter in CV
$= -F$

$$\therefore \quad \boxed{F = \dot{m}V(1-\cos\theta)} \qquad \left(\dot{m} = \frac{\pi}{4}\rho V^2 d^2\right)$$

−5 Minor confusion on otherwise correct balance (eg 2 vs 3 dimensions; dimensional slips)

END