George M. Bergman
5 Evans Hall

Fall 2002, Math 113, Sec. 5

First Midterm

30 Sept., 2002
3:10-4:00

1. (21 points, 7 points each.) Find the following.

(a) An expression for $(1\ 2\ 3\ 4\ 5\ 6)^2$ as a product of disjoint cycles in S_7.

(b) The g.c.d. (3399, 9785).

(c) The integer r such that $0 \leq r < 13$ and $95^{13} \equiv r \bmod 13$.

2. (28 points; 7 points each.) Complete the following definitions. (In each definition, you can use without defining them any terms or symbols that were defined in the text before that definition.)

(a) Given integers a, b and m, we write $\boldsymbol{a \equiv b \bmod m}$ if ...

(b) Given a function $f: X \to Y$, an ***inverse*** to f means a function g with domain ____ and target ____, such that ...

(c) If r and n are positive integers, a permutation $\sigma \in S_n$ is called an ***r-cycle*** if ...

(d) A group G with operation $*$ is called ***abelian*** if ...

3. (24 points; 7 points each.) For each of the items listed below, either *give an example,* or give a brief reason why *no example exists.* (If you give an example, you do *not* have to prove that it has the property stated.)

(a) Two positive integers a and b, neither of which is divisible by 8, but whose product is divisible by 8.

(b) Sets X, Y and Z, and functions $f: X \to Y$, $g: Y \to X$, such that $g \circ f$ is bijective, but f is not.

(c) Transpositions $\tau_1, \tau_2, \tau_3 \in S_4$ such that $(1\ 2\ 3) = \tau_1 \tau_2 \tau_3$.

4 (27 points). Prove the following result from the text. You may assume in your proof results proved in the text before it, but not results proved after it.

Theorem (Euclid's Lemma). *If p is a prime and a, b are integers such that $p \mid ab$, then $p \mid a$ or $p \mid b$.*