

Math H113

Professor K. A. Ribet

1st Midterm Exam

February 20, 2003

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. Please write your name on each sheet of paper that you turn in; don't trust staples to keep your papers together. Explain your answers in full English sentences as is customary and appropriate. Your paper is your ambassador when it is graded.

1. (5 points) Find integers x and y that satisfy $47x + 63y = 1$.
2. (6 points) Let N_1 and N_2 be normal subgroups of a group G . Suppose that x is an element of N_1 and that y is an element of N_2 . Show that $xyx^{-1}y^{-1}$ belongs to the intersection $N_1 \cap N_2$.
3. (9 points) Let H be a subgroup of a finite group G . Recall that the normalizer $N_G(H)$ of H is the group of all $g \in G$ such that $gHg^{-1} = H$. Show that the number of distinct subgroups gHg^{-1} (with $g \in G$) is the index $(G : N_G(H))$ and that this number divides $(G : H)$. If H is smaller than G , show that the union of the gHg^{-1} is smaller than G .
4. (5 points) Consider the products $\sigma = (123)(45)(243)$, $\tau = (243)(45)(123)$ in the symmetric group \mathbf{S}_5 . Write each as a product of disjoint cycles. Show that σ and τ are conjugate in \mathbf{S}_5 .
5. (5 points) Let G be a group with the following property: if two elements of G commute with each other, either the elements are equal or at least one of them is the identity. Show that G has at most 2 elements.