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Spring 1997, Math 113, Lecture 3 — 4 April, 1997

Second Midterm Exam — 1:10-2:00 PM

1. (30 points) Mark statements ***T*** (true) or ***F*** (false). A meaningless statement should be considered false. Each correct answer will count 2 points, each incorrect answer −2 points, each unanswered item 0 points.

____ Every group of order 11 is abelian.

____ S_4 has an element of order 8.

____ If G and G' are abelian groups, so is the product group $G \times G'$.

____ If a product group $G \times G'$ is abelian, so are G and G'.

____ Every homomorphism of groups is a one-to-one map.

____ If G is a group, and $a \in G$, then the map $\varphi_a\colon G \to G$ defined by $\varphi_a(g) = ag$ is an automorphism of G.

____ If G is a group, and $a \in G$, then the map $\varphi_a\colon G \to G$ defined by $\varphi_a(g) = aga^{-1}$ is an automorphism of G.

____ The kernel of every group homomorphism $\varphi\colon G \to G'$ is a subgroup of G.

____ The image of every group homomorphism $\varphi\colon G \to G'$ is a normal subgroup of G'.

____ Every group is isomorphic to a subgroup of the group of permutations of some set.

____ A_8 is a simple group.

____ $A_7 \times A_7$ is a simple group.

____ S_{11} is a simple group.

____ If x is an element of a G-set X, then the orbit Gx and the isotropy subgroup G_x are isomorphic to each other.

____ If G is a group and X a set, and for each $g \in G$ and $x \in X$ we define $gx = x$, this makes X a G-set.

2. (30 points) Suppose G is a group, H a subgroup, and g_1, g_2 elements of G. Show that $g_1 H = g_2 H$ if and only if $g_1^{-1} g_2 \in H$. (You may use any results proved in the reading, if you state clearly what the results you are calling on are.)

3. (25 points) Let G be a group, let X be a G-set, and let $x \in X$. We recall that G_x is defined to be $\{g \in G \mid gx = x\}$, and called the "isotropy subgroup of x".

Prove that G_x is indeed a subgroup of G.

4. (15 points) Suppose G is a group and X and Y are two G-sets. Let us make the set $X \times Y$ a G-set by defining $g(x,y) = (gx, gy)$ for $x \in X$, $y \in Y$, $g \in G$. (Take for granted that this *does* determine a structure of G-set on $X \times Y$; i.e., you are not being asked to prove this.)

For any $x \in X$ and $y \in Y$, show how $G_{(x,y)}$ (the isotropy subgroup of the element $(x,y) \in X \times Y$) can be described in terms of the subgroups G_x and G_y. Show your reasoning.