Math H113 Professor K. A. Ribet
2nd Midterm Exam April 8, 2003

Please put away all books, calculators, electronic games, cell phones, pagers, .mp3 players, PDAs, and other electronic devices. Please write your name on each sheet of paper that you turn in; don't trust staples to keep your papers together. Explain your answers in full English sentences as is customary and appropriate. Your paper is your ambassador when it is graded.

1. (4 points) Show that a group of order 40 has a normal subgroup of order different from 1 and 40.

2. (5 points) Find the number of elements of order 7 in a simple group of order $168 = 2^3 \cdot 3 \cdot 7$.

3. (7 points) Let G be a finite group of p-power order. Let N be a normal subgroup of G of order p. Prove that N is contained in the center of G.

4. (7 points) Consider the cycle $\sigma = (1\,2 \cdots n-1\,n)$ in S_n. Show that σ has $(n-1)!$ conjugates and that the centralizer of σ has order n.

5. (7 points) Let S and T be subsets of a finite group G. Suppose that $|S| + |T| > |G|$. Show, for each $g \in G$, that the intersection of S with $gT^{-1} = \{\, gt^{-1} \mid t \in T \,\}$ is non-empty. Prove that G coincides with its subset $ST = \{\, st \mid s \in S, t \in T \,\}$.