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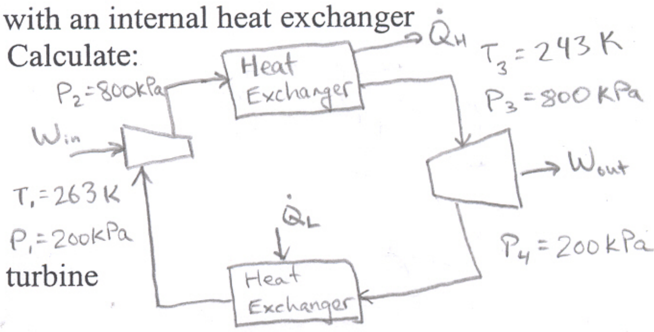
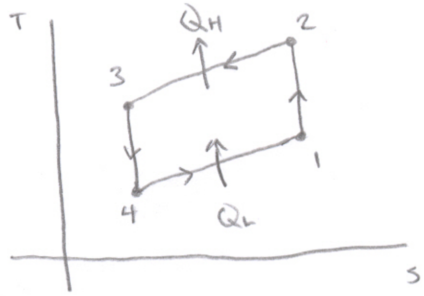
Question 1: [25 points]

Before the big game, Cal M.E. students were freezing Stanford's footballs before kick-off with a gas refrigeration system (this did not affect the outcome of the game). Air enters the isentropic compressor of a gas refrigeration cycle at -10 degrees C and is compressed from 200 kPa to 800 kPa. The high-pressure air is then cooled to -30 degrees C with an internal heat exchanger before it enters the turbine. **Draw this on a T-s diagram.** Calculate:



$k = 1.4$
$c_p = 1.005$ kJ/kg-K
Assume ideal components.

a) the minimum possible temperature of the air leaving the turbine
b) q_{in}
c) w_{net}
d) the coefficient of performance



a) Min. exit temperature

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = (243\text{K})\left(\frac{200}{800}\right)^{\frac{1.4-1}{1.4}} \rightarrow \boxed{T_4 = 163.5\text{ K}}$$

b) q_{in}:

$$q_{in} = C_p(T_1 - T_4) = \left(1.005 \tfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(263 - 163.5\text{K}) \rightarrow \boxed{q_{in} = 100 \tfrac{\text{kJ}}{\text{kg}}}$$

c) W_{net}:

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 390.8\text{ K}$$

$$\text{Turbine } W_{out} = h_3 - h_4 = C_p(T_3 - T_4) = \left(1.005 \tfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(243 - 163.5\text{K}) = 80 \tfrac{\text{kJ}}{\text{kg}}$$

$$\text{Compressor } W_{in} = h_2 - h_1 = C_p(T_2 - T_1) = \left(1.005 \tfrac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(390.8 - 263\text{K}) = 128.4 \tfrac{\text{kJ}}{\text{kg}}$$

$$W_{net} = W_{in} - W_{out} \Rightarrow \boxed{W_{net} = 48.4 \tfrac{\text{kJ}}{\text{kg}} \text{ [into system]}}$$

d) $$COP_R = \frac{q_{in}}{W_{net}} = \frac{100}{48.4} \rightarrow \boxed{COP_R = 2.06}$$

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Question 2: [20 points]

During the Big Game, it was clear that Stanford was lost and confused in their play-calling (thus generating more entropy than the Berkeley team). Derive a general relation for differential entropy changes (ds) in terms of c_v, v, T, and P. Show each step in your derivation.

$$s(T,v): \quad ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \quad (1)$$

$$u(T,v): \quad du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$= C_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv \quad (2)$$

Gibb's Relation: $du = Tds - Pdv \quad (3)$

(1) and (2) into (3)

$$C_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv = T\left[\left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv\right] - Pdv$$

Equating dT coefficients: $\left(\frac{\partial s}{\partial T}\right)_v = \frac{C_v}{T} \quad (4)$

From Maxwell Relations: $\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial P}{\partial T}\right)_v \quad (5)$

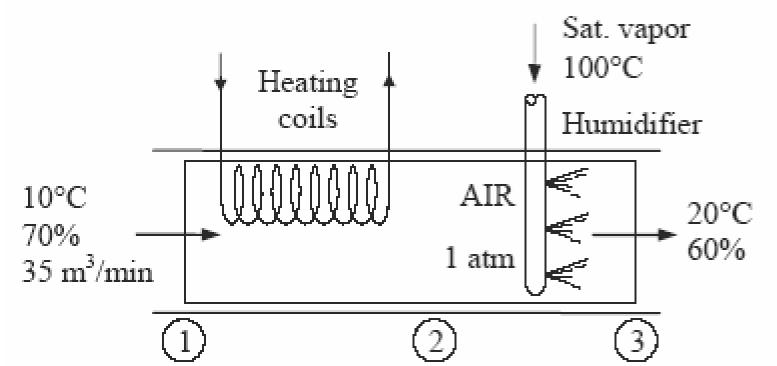
(4) and (5) into (1):

$$\boxed{ds = \frac{C_v}{T} dT + \left(\frac{\partial P}{\partial T}\right)_v dv}$$

Question 3: [25 points]

The Stanford team, not being used to practicing outside their air-conditioned practice facility was stricken by the scorching 70ºF weather on Saturday. Let's analyze their air conditioning unit:

The air-conditioning system operates at a total pressure of 1 atm and consists of a heating section and a humidifier that supplies wet steam (saturated water vapor) at 100 degrees C. Air enters the heating section at 10 degrees C and 70 percent relative humidity at a rate of 35 m^3/min, and it leaves the humidifying section at 20 degrees C and 60 percent relative humidity. Determine:



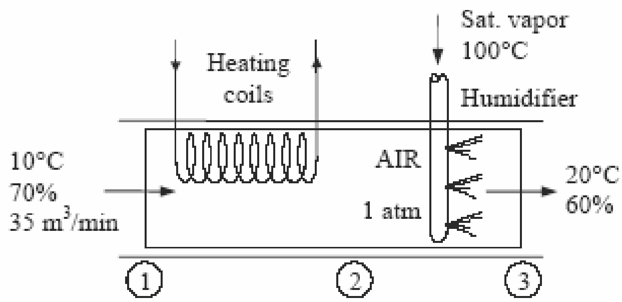
a) the **enthalpy and specific humidity** at states **one** and **three** and the **specific volume** at state **one** (using the psychrometric chart)
b) the enthalpy at state two
c) the temperature and relative humidity of air when it leaves the heating section
d) the rate of heat transfer in the heating section
e) the rate at which water is added to the air in the humidifying section

14-80 Air is first heated and then humidified by wet steam. The temperature and relative humidity of air at the exit of heating section, the rate of heat transfer, and the rate at which water is added to the air are to be determined.

Assumptions **1** This is a steady-flow process and thus the mass flow rate of dry air remains constant during the entire process $(\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a)$. **2** Dry air and water vapor are ideal gases. **3** The kinetic and potential energy changes are negligible.

Properties The inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. The properties of the air at various states are determined from the psychrometric chart (Figure A-31) to be

$$h_1 = 23.5 \text{ kJ/kg dry air}$$
$$\omega_1 = 0.0053 \text{ kg H}_2\text{O/kg dry air } (= \omega_2)$$
$$v_1 = 0.809 \text{ m}^3\text{/kg dry air}$$
$$h_3 = 42.3 \text{ kJ/kg dry air}$$
$$\omega_3 = 0.0087 \text{ kg H}_2\text{O/kg dry air}$$



Analysis (*a*) The amount of moisture in the air remains constant it flows through the heating section ($\omega_1 = \omega_2$), but increases in the humidifying section ($\omega_3 > \omega_2$). The mass flow rate of dry air is

$$\dot{m}_a = \frac{\dot{V}_1}{v_1} = \frac{35 \text{ m}^3\text{ / min}}{0.809 \text{ m}^3\text{ / kg}} = 43.3 \text{ kg/min}$$

Noting that $Q = W = 0$, the energy balance on the humidifying section can be expressed as

$$\dot{E}_{in} - \dot{E}_{out} = \Delta\dot{E}_{system}^{\nearrow 0 \text{ (steady)}} = 0$$
$$\dot{E}_{in} = \dot{E}_{out}$$
$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \longrightarrow \dot{m}_w h_w + \dot{m}_{a2} h_2 = \dot{m}_a h_3$$
$$(\omega_3 - \omega_2) h_w + h_2 = h_3$$

Solving for h_2,

$$h_2 = h_3 - (\omega_3 - \omega_2) h_{g@100°C} = 42.3 - (0.0087 - 0.0053)(2675.6) = 33.2 \text{ kJ/kg dry air}$$

Thus at the exit of the heating section we have $\omega_2 = 0.0053$ kg H_2O dry air and $h_2 = 33.2$ kJ/kg dry air, which completely fixes the state. Then from the psychrometric chart we read

$$T_2 = \mathbf{19.5°C}$$
$$\phi_2 = \mathbf{37.8\%}$$

(*b*) The rate of heat transfer to the air in the heating section is

$$\dot{Q}_{in} = \dot{m}_a (h_2 - h_1) = (43.3 \text{ kg/min})(33.2 - 23.5) \text{ kJ/kg} = \mathbf{420 \text{ kJ/min}}$$

(*c*) The amount of water added to the air in the humidifying section is determined from the conservation of mass equation of water in the humidifying section,

$$\dot{m}_w = \dot{m}_a (\omega_3 - \omega_2) = (43.3 \text{ kg/min})(0.0087 - 0.0053) = \mathbf{0.15 \text{ kg/min}}$$

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Question 4: [25 points]

At a pre-game tea-party, Stanford M.E. students were puzzled as to the nature of water vaporization. At an atmospheric pressure of $P_1 = 101.3$ kPa, water boils at T_1=100°C, the internal energy of vaporization is $U_{fg} = 2087$kJ/kg. At these conditions, water has a specific volume of $v_1 = 1.6720$ m^3/kg estimate the temperature T_2 when $P_2 = 50$ kPa.

$M_{H2O} = 18$ kg/kmol
$R_u = 8.314$ kJ/kmol·K

Water boiling temp: $P_1 = 101.3$ kPa $v_1 = 1.6720$ m^3/kg
$T_1 = 100°C$ $U_{fg} = 2087$ kJ/kg

$M_{H_2O} = 18$ kg/kmol $R = \dfrac{8.314 \frac{kJ}{kmol \cdot K}}{18 \text{ kg/kmol}} = 0.462 \frac{kJ}{kg \cdot K}$

We must estimate T_2 when $P_2 = 50$ kPa

$$h_{fg} = U_{fg} + Pv$$
$$= 2087 \frac{kJ}{kg} + (101.3 \text{ kPa})(1.6720 \frac{m^3}{kg})$$
$$h_{fg} = 2256.4 \frac{kJ}{kg}$$

Using Clausius-Clapeyron Equation:

$$\ln\left(\frac{P_2}{P_1}\right)_{sat} = \frac{h_{fg}}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)_{sat}$$

Solving for T_2 → $\boxed{T_2 = 354 \text{ K}}$

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Question ~~6~~ 5: [5 points]

During the Stanford pre-game tea-party, a group of students were discussing partial differential equations. They particularly had trouble with these questions:

6A....If dz = Mdx + Ndy, with M=3 and N=7 what is $\left(\frac{\partial z}{\partial x}\right)_y$?

$$\left(\partial z / \partial x\right)_y = 3$$

6B....If $\left(\frac{\partial N}{\partial x}\right)_y = 9$, what is $\left(\frac{\partial M}{\partial y}\right)_x$?

$$\left(\partial M / \partial y\right)_x = 9$$