Midterm #1 Solutions and
Grading Key

1) *(20 Points)* Analyzing the interior of the piston cylinder as the system, we brake the process into two sub-processes: (1) initially at rest (position a) to just as the piston raises off the lower blocks (position b) and (2) just as the piston raises off the lower blocks (position b) to just before striking the upper blocks (position c). We use the first law for a closed system process to begin the problem:

$$\partial Q_{ac} = dE_{ac} + \partial W_{ac}$$
$$\partial Q_{ac} = m_{air}\left(u_c - u_a\right) + \partial W_{ac}$$
$$\partial Q_{ac} = m_{air} C_v \left(T_c - T_a\right) + \partial W_{ac}$$

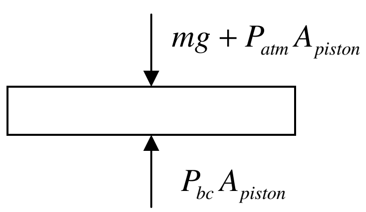
- 3 points for proper first law
- 2 points for correct reduction of dE

We now consider the work term, separating it according to the sub-processes described above.

$$\partial W_{ac} = \partial W_{ab} + \partial W_{bc}$$
$$\partial W_{ac} = \int_a^b PdV + \int_b^c PdV$$
$$\partial W_{ac} = P_{bc}\left(V_c - V_b\right)$$

- 2 point for correctly separating the problem into a constant volume process and a constant pressure process
- 3 points for the correct equation for the work

We must now find the pressure of the air during process b to c. A force balance on the piston yields the P_{bc}.



$$mg + P_{atm} A_{piston} = P_{bc} A_{piston}$$
$$P_{bc} = \frac{mg}{A_{piston}} + P_{atm}$$
$$P_{bc} = \left(\frac{25kg \cdot 9.8m/s^2}{0.0050m^2}\right)\left(\frac{1kPa}{1000Pa}\right) + 101kPa$$
$$P_{bc} = 150kPa$$

- 2 point for producing the proper force balance
- 1 point for 150 kPa

We now calculate the change in volume between positions b and c.

$$\left(V_c - V_b\right) = \Delta h_{bc} A_{piston} = 0.1m \cdot 0.0050m^2 = 0.0005m^3$$

We can now find the word done during the process.

$$\partial W_{ac} = P_{bc}\left(V_c - V_b\right) = 150kPa \cdot 0.0005m^3 = 0.075kJ$$

- 2 point for correctly finding 0.075 kJ for work

Returning to the first law to find the heat transfer, we must find the mass of the air in the piston-cylinder. The ideal gas law applied at position a yields the mass.

$$m = \frac{P_a V_a}{R_{air} T_a} = \frac{101kPa \cdot 0.0050m^2 \cdot 0.25m}{0.287kJ/kgK \cdot 293K} = 0.0015kg$$

- 1 point for correctly applying the ideal gas law at position a
- 1 point for finding m = 0.0015 kg

We now need to find the temperature at position c. Again, the ideal gas law should be applied – this time at position c.

$$T_c = \frac{P_{bc} V_c}{R_{air} m} = \frac{150kPa \cdot 0.0050m^2 \cdot 0.35m}{0.287kJ/kgK \cdot 0.0015kg} = 609.76K$$

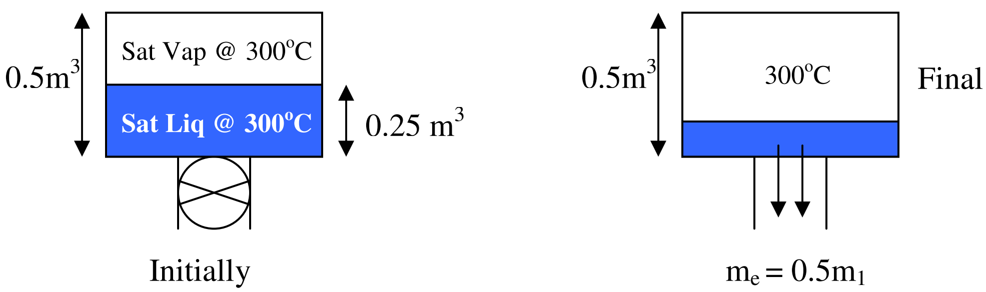
- 1 point for finding T_c = 609.7 K

Finally, the heat transfer can be calculated.

$$\partial Q_{ac} = m_{air} C_v (T_c - T_a) + \partial W_{ac}$$
$$\partial Q_{ac} = 0.0015kg \cdot 0.72kJ/kgK \cdot (609.76 - 293)K + 0.075kJ$$
$$\partial Q_{ac} = 0.417kJ$$

- 2 point for finding HX = 0.417 kJ

2) *(20 points)* We begin by defining our system as the vessel and sketching the process.



This is a transient, open system. Thus we must use the general form of the first law:

$$Q_{CV} + \sum_i m_i \left(h_i + \frac{V_i^2}{2} + gZ_i \right) = dE_{cv} + W_{CV} + \sum_e m_e \left(h_e + \frac{V_e^2}{2} + gZ_e \right)$$
$$Q_{CV} = m_2 u_2 - m_1 u_1 + m_e h_e = m_1 (0.5u_2 - u_1 + 0.5h_e)$$

- 3 points for correct form of the first law
- 3 points for reducing first law to last form shown

We continue by finding the mass at the initial time (m_1).

$$m_1 = m_{liquid,1} + m_{vapor,1} = \frac{V_{liquid,1}}{v_f\ @\ 300^o C} + \frac{V_{vapor,1}}{v_g\ @\ 300^o C}$$

$$m_1 = \frac{0.25m^3}{0.001404m^3/kg} + \frac{0.25m^3}{0.02167m^3/kg} = 178.1kg + 11.54kg = 189.6kg$$

- 2 points for proper method in finding m_1
- 1 point for finding m_1 = 189kg

We now calculate x_1 and u_1.

$$x_1 = \frac{m_{vapor,1}}{m_1} = \frac{11.54kg}{189.6kg} = 0.061$$

$$u_1 = u_{f\ @\ 300^o C} + xu_{fg\ @\ 300^o c} = 1332.0kJ/kg + 0.061 \cdot 1231.0kJ/kg$$

$$u_1 = 1406.9kJ/kg$$

- 2 point for recognizing 1 is a mixture
- 1 point for finding x_1 = 0.061
- 1 point for u_1=1406.9kJ/kg

We must now evaluate u_2. We do so by recognizing that $m_2 = 0.5m_1$ and that $V_1=V_2$.

$$v_2 = \frac{V_2}{m_2} = \frac{0.5m^3}{0.5 \cdot 189.6kg} = \frac{0.5m^3}{94.8kg} = 0.005264m^3/kg$$

$$Since\quad v_{f\ @\ 300^o C} < v_2 < v_{g\ @\ 300^o C} \rightarrow Still\ a\ mixture$$

$$x_2 = \frac{v_2 - v_{f\ @\ 300^o C}}{v_{fg\ @\ 300^o C}} = \frac{0.005264 - 0.001404}{0.02167 - 0.001404} = 0.191$$

$$u_2 = u_{f\ @\ 300^o C} + xu_{fg\ @\ 300^o c} = 1332.0kJ/kg + 0.191 \cdot 1231.0kJ/kg$$

$$u_2 = 1567.1kJ/kg$$

- 1 point for recognizing 2 is a mixture
- 1 point for u_2=1567.1kJ/kg

Realizing that only the saturated liquid is draining off the bottom of the vessel, we can find h_e.

$$h_e = h_{f\ @\ 300^o C} = 1344.0kJ/kg$$

- 3 points for recognizing $h_e = h_{f@300^oC}$

Finally, we can use the first law to find the added heat.

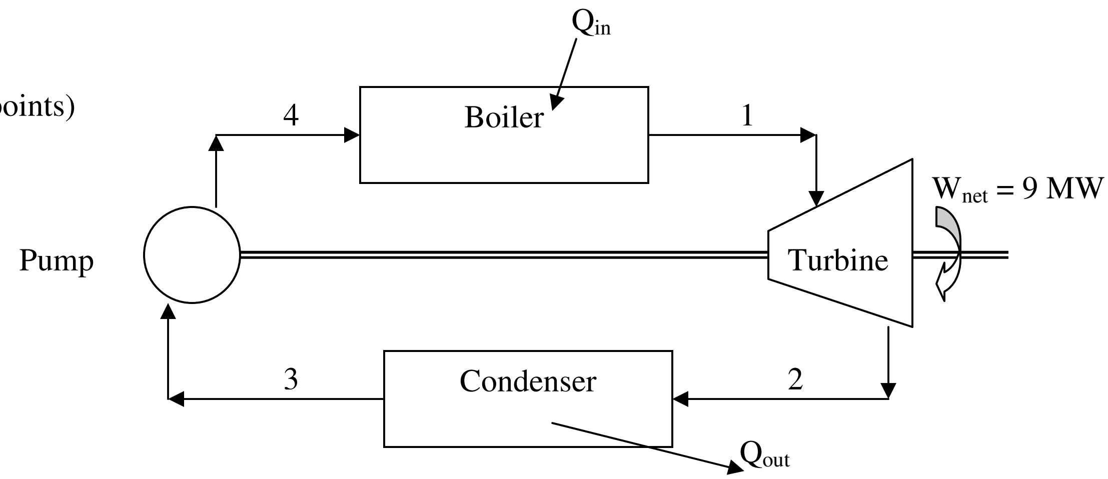
$$Q_{CV} = m_1(0.5u_2 - u_1 + 0.5h_e)$$

$$Q_{CV} = 189.6kg(0.5 \cdot 1567.1 - 1406.9 + 0.5 \cdot 1344.0)kJ/kg$$

$$Q_{CV} = 9224kJ$$

- 2 points for Q_{cv} = 9224 kJ

3) (20 points)



We begin by recognizing that the net work output is given by the following expression.

$$\dot{W}_{net} = \sum_{Cycle} \dot{W} = \dot{W}_{turbine} + \dot{W}_{pump}$$

- 2 points for the definition of W_{net}

We now use the steady state, steady flow form of the first law to analyze the adiabatic turbine and pump.

$$\dot{m}h_1 = \dot{m}h_2 + \dot{W}_{turbine} \rightarrow \dot{W}_{turbine} = \dot{m}(h_1 - h_2)$$
$$\dot{m}h_3 = \dot{m}h_4 + \dot{W}_{pump} \rightarrow \dot{W}_{pump} = \dot{m}(h_3 - h_4)$$

- 4 points for steady state analysis of turbine and pump

Substituting these equations into the definition of net work, we can find the mass flow rate.

$$\dot{W}_{net} = \dot{W}_{turbine} + \dot{W}_{pump} = \dot{m}(h_1 - h_2) + \dot{m}(h_3 - h_4)$$
$$\dot{m} = \frac{\dot{W}_{net}}{h_1 - h_2 + h_3 - h_4}$$

- 2 points for correct mass flow rate equation

We must now evaluate the enthalpies at positions 1, 2, 3 and 4.

$$\left.\begin{array}{l} P_1 = 10MPa \\ T_1 = 550^o C \end{array}\right\} h_1 = 3500.9 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_2 = 0.010MPa \\ x_2 = 0.86 \end{array}\right\} \begin{array}{l} h_2 = h_{f\,@10kPa} + x h_{fg\,@10kPa} \\ h_2 = 191.83 kJ/kg + 0.86 \cdot 2392.8 kJ/kg = 2249.6 kJ/kg \end{array}$$

$$\left.\begin{array}{l} P_3 = 0.010MPa \\ Sat.\ Liquid \end{array}\right\} h_3 = h_{f\,@\,0.01MPa} = 191.83 \text{ kJ/kg}$$

$$P_4 = 10MPa$$
$$T_4 = T_{sat\,@\,10kPa} = 45.81^oC$$

$$h_4 = h_{f\,@\,45.81^oC} + v_{f\,@\,45.81^oC}(P_4 - P_{sat\,@\,45.81^oC})$$
$$h_4 = 191.83kJ/kg + 0.001010m^3/kg \cdot (10 - 0.010)x10^3 kPa$$
$$h_4 = 201.92kJ/kg$$

- 2 points for h_4

We can now find the mass flow rate.

$$\dot{m} = \frac{\dot{W}_{net}}{h_1 - h_2 + h_3 - h_4} = \frac{9000kW}{(3500.9 - 2249.6 + 191.83 - 201.92)kJ/kg} = 7.25kg/s$$

- 1 point for m_dot

Using the relationships above, we can find the work of the turbine and the pump.

$$\dot{W}_{turbine} = \dot{m}(h_1 - h_2) = 7.25kg/s(3500.9 - 2249.6)kJ/kg = 9.07MW$$
$$\dot{W}_{pump} = \dot{m}(h_3 - h_4) = 7.25kg/s(191.83 - 201.92)kJ/kg = -73.2kW$$

- 2 points

The percent of heat input that is converted to net work is given by:

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}in}$$

- 2 points

The rate of heat input is given by a first law analysis of the boiler.

$$\dot{Q}_{in} = \dot{m}(h_1 - h_4) = 7.25kg/s(3500.9 - 201.92)kJ/kg = 23.92MW$$

- 2 points

Thus:

$$\eta = \frac{\dot{W}_{net}}{\dot{Q}in} = \frac{9MW}{23.92MW} = 0.376 \text{ or } 37.6\%$$

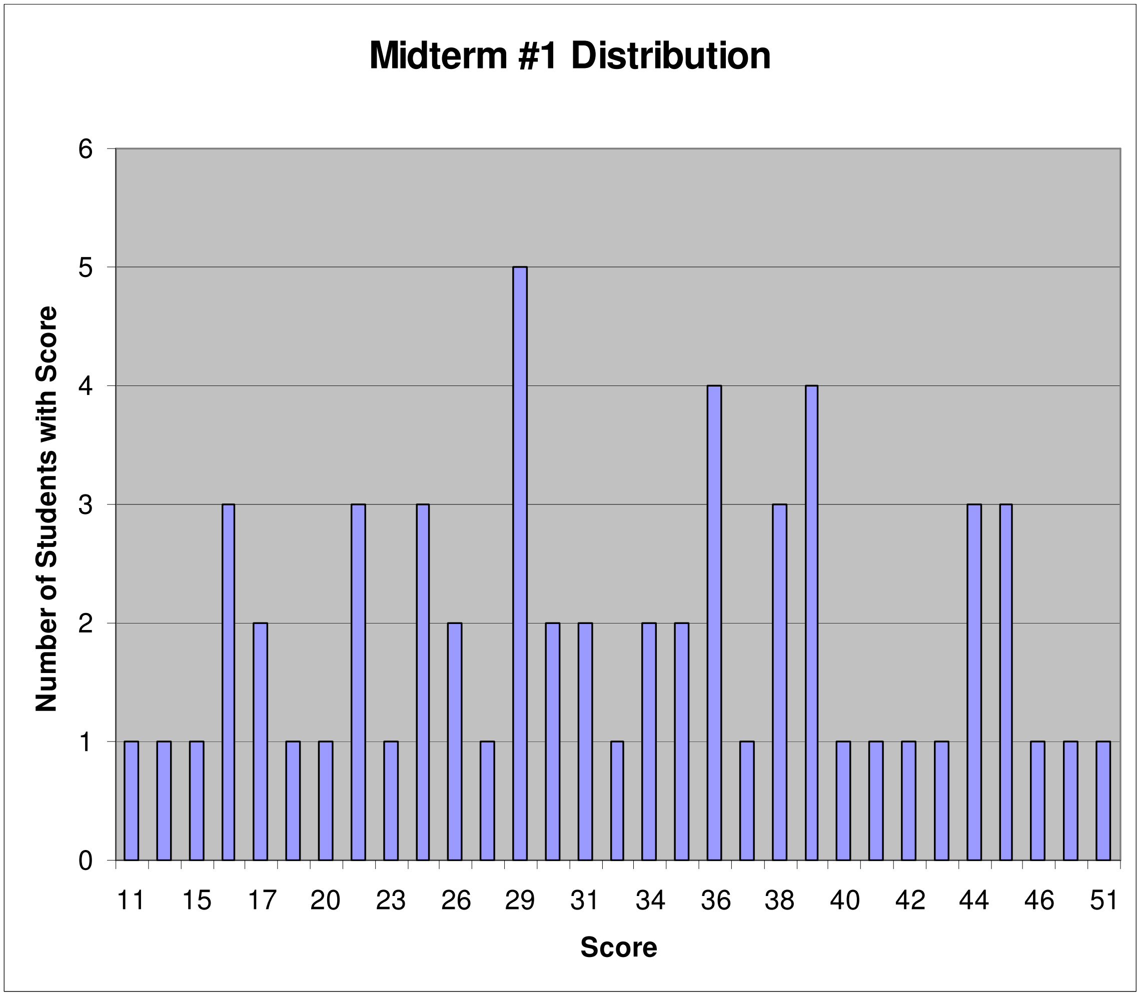
- 1 point

The net rate of heat output by the cycle is given by the first law for a cycle or by a first law analysis of the condenser.

$$\dot{Q}_{out} = \dot{Q}_{in} - \dot{W}_{net} = \dot{m}(h_2 - h_3) = 23.92MW - 9MW = 14.92MW$$

- 2 points

Results:



Overall Average: 31.2 out of 60
Standard Deviation: 10.8

Average for Problem #1: 14.6
Average for Problem #2: 10.8
Average for Problem #3: 6.3