Department of Mathematics, University of California, Berkeley

Math 214

Alan Weinstein, Fall 2002

Take Home Final Examination

Due in class at 11:15 AM, Tuesday, 12/3/02 (GROUP 1), or in
825 Evans Hall (under the door) by 11:15 AM, Thursday, 12/12/02 (GROUP 2).

Instructions. You may use your class notes and the Spivak text, but no other references. You should consult nobody except A.W. about the exam. Please send questions about the exam to `alanw@math.berkeley.edu` and not to the course emailing list. If I learn of errors or imprecisions on the exam, I will send corrections to members of Group 1 who send me an email request to be placed on the mailing list for such announcements. Once the exam has been distributed to Group 2, I will send corrections to the emailing list (and they will then appear at `http://socrates.berkeley.edu/~alanw/mail-archive.214` as well).

Do all of the problems. If you have trouble with one part of a problem, you may still use its result to try the following parts. Unless otherwise specified, all manifolds, maps, flows, actions, ... are C^∞.

1. A 1-form α on a manifold M of dimension $2n+1$ is called a **contact form** if $\alpha \wedge (d\alpha)^n$ is a nowhere vanishing $2n+1$ form.

A. Show that, if α is a contact form on M, then there is a unique vector field X on M such that $X \lrcorner \alpha = 1$ and $X \lrcorner d\alpha = 0$. X is called the **Reeb vector field** of α.

B. Let ϕ_t be the flow (possibly locally defined) of the Reeb vector field of α. Prove that α is invariant under this flow; i.e. $\phi_t^*(\alpha) = \alpha$ for all t.

C. Show that if f is a nowhere-vanishing function on M, and if α is a contact form, then so is $f\alpha$.

D. The **kernel** of a contact form α is the distribution on M consisting of those tangent vectors v for which $v \lrcorner \alpha = 0$. A **contact structure** on M is a distribution $F \subset TM$ which is locally defined by contact forms; i.e. each p in M is contained in an open set $\mathcal{U}$ on which there is a contact form whose kernel is the restriction of F to $\mathcal{U}$. Prove that a contact structure F is globally defined by a contact form, i.e. F is the kernel of a contact form α on M, if and only if the quotient vector bundle TM/F is trivial.

E. Let M be the sphere of radius r with center at the origin in $\mathbb{R}^4$. Let F be the distribution consisting of those vectors tangent to M which are orthogonal (with the standard euclidean metric) to the vector field

$$x_1\frac{\partial}{\partial x_2} - x_2\frac{\partial}{\partial x_1} + x_3\frac{\partial}{\partial x_4} - x_4\frac{\partial}{\partial x_3}.$$