MATH 170 J. Mihalisin

FINAL EXAMINATION

AUTUMN 2002

Your Name

Your Signature

Problem	Total Points	Score
1	10	
2	20	
3	20	
4	30	
5	30	
6	30	
Total	140	

- You may use both sides of one $8\frac{1}{2} \times 11$ sheet of notes (your "cheat sheet").
- **If any of the problem statements are unclear then please ask!! (I do not intend to test you on comprehension of confusing statements.)**
- **If you use a result proved in class or in the text, you must CLEARLY state what you are doing.**
- **Raise your hand if you have a question.**

MATH 170 **FINAL EXAMINATION** AUTUMN 2002

1 (10 points)

Within the simplex method, exactly what can you infer if every negative reduced costs corresponds to an infeasible direction?

MATH 170 **FINAL EXAMINATION** AUTUMN 2002

2 (20 points)

Consider the zero-sum game represented by matrix G, using the standard conventions that the rows represent Player 1's moves, the columns represent Player 2's moves and that a positively valued outcome is favorable for Player 1.

Assume there exists some row vector $\vec{x}$ with:

$$\vec{x} \cdot G = (\alpha, \beta, \alpha, \alpha, \beta) \text{ and } \beta < \alpha.$$

Further, assume there exist a column vector $\vec{y}$ with:

$$G \cdot \vec{y} = (\gamma, \delta, \gamma, \delta, \gamma)^T \text{ and } \delta < \gamma.$$

Note: $\vec{x}$ and $\vec{y}$ are not assumed to be optimal mixed strategies.

$\vec{x}, \vec{y} \geq 0$

$\sum x_i \cdot \sum y_i = 1$

a) What can you infer (if anything) about the relative order of α, β, γ and δ? Why?

b) What can you infer (if anything) about the value of the game? Why?

3 (20 points)

Given the following linear program and its dual –

minimize $\vec{c} \cdot \vec{x}$
subject to:
$A \cdot \vec{x} \geq \vec{b}$
$\vec{x} \geq \vec{0}$

maximize $\vec{p} \cdot \vec{b}$
subject to:
$A^T \cdot \vec{p} \leq \vec{c}$
$\vec{p} \geq \vec{0}$

Prove that the coordinates for the optimal solutions x^* and p^* can be inferred from the optimal strategy for the following game:

$$G = \begin{pmatrix} 0 & -A & +\vec{b} \\ A^T & 0 & -\vec{c} \\ -\vec{b}^T & \vec{c}^T & 0 \end{pmatrix}$$

(Hint: G is an $m+n+1$ by $m+n+1$ matrix. The first $m+n$ components of the optimal strategy vector are scaled multiples of the coordinates of x^* and p^*, the final coordinate of the strategy compensates for this scaling by multiplying $\vec{b}$ and $\vec{c}$.)

MATH 170 **FINAL EXAMINATION** AUTUMN 2002

4 (30 points)

Do JUST ONE of the following two problems:
(Cross out the one you DON'T attempt)

(I).

Describe the Simplex algorithm in the context of network flow problems.
In particular:

(a) What do the basic solutions look like? (Justify your answer.)

(b) How does an iteration of the Simplex method operate on the solution?

(c) What does a change of basis look like?

(II).

Consider an arbitrary directed graph with an origin node S and a destination node T.

We define the *connectivity* of the graph to be the maximum number of directed paths from S to T that do not share any nodes (except for S and T).

We define the *vulnerability* of the graph as the minimum number of nodes (besides S or T) that need to be removed so that there exists no directed path from s to t.

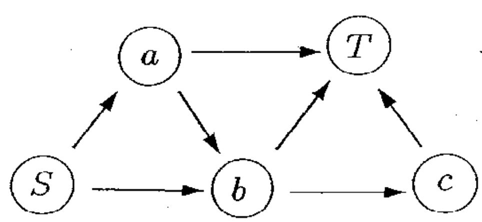
Prove that for any directed graph the connectivity equals the vulnerability.
(Hint: Convert the connectivity problem into a maximum flow problem.)

For example:

Connectivity = 2, consider:

$S > a > T$

$S > b > c > T$



Vulnerability = 2:

can remove a and b

(The next page is blank to give room for your answer.)

5 (30 points)
Do JUST ONE of the following two problems:
(Cross out the one you DON'T attempt)

(I).

A typesetting program (such as TeX) determines the page breaks as follows:

(i) The document consists of a sequence of n items.

(ii) A page starting with item i and ending with item j is assigned an attractiveness factor c_{ij}.

With all the c_{ij} as known quantities, the page breaks are chosen to maximize the sum of the attractiveness factors.

a) Formulate the problem as an Integer Program.

b) Devise a dynamic programming algorithm to solve the problem.

(II).

Using the conventions of section 11.4, we form the Lagrangean dual of an integer program by splitting the constraints into two sets:

minimize $\vec{c} \cdot \vec{x}$

subject to:

$$A \cdot \vec{x} \geq \vec{b}$$
$$D \cdot \vec{x} \geq \vec{d}$$
$$\vec{x} \text{ integer}$$

The Lagrangean dual is a new problem where the integrality as it relates to the A constraints have been relaxed in some sense. Its minimum is called Z_D.

For a given IP, consider two different relaxations – the first separates the IP's constraints into A and D, the second separates them into A' and D'.

If A contains all the constraints of A' (i.e. if the first problem relaxes all the constraints relaxed in the second problem and possibly more), prove that $Z_D \leq Z_{D'}$.

(The next page is blank to give room for your answer.)

6 (30 points)

Do JUST ONE of the following two problems:
(Cross out the one you DON'T attempt)

(I).

In as much detail as possible (e.g. sketches of proofs), describe the role volume plays in the ellipsoid method and list the various results relevant for that role.

(I.e., give qualitative results and prooflets for Lemma 8.2, Lemma 8.3 and Lemma 8.4.)

(II).

In as much detail as possible, describe the ideas and methods behind the "primal path following algorithm".

(The next page is blank to give room for your answer.)