Fall 2001 Second Midterm R. Borcherds

Math 121A midterm, Tuesday October 30, 9:40-11:00.

Please make sure that your name is on everything you hand in.
You are allowed calculators and 1 page of notes.
All questions have about the same number of marks.

1. Find the point on the sphere $x^2+y^2+z^2=1$ for which $x+2y+2z$ is a maximum.

2. If $w=\int_x^{x^2} e^{-u^2}du$ find dw/dx.

3. Show that $x/(x^2+y^2)$ is harmonic (in other words, it satisfies Laplace's equation). Find an analytic function $f(z)=f(x+iy)$ of which it is the real part. (Hint: the function has degree -1, so the same is likely to be true for f.) Find the conjugate harmonic function $Im(f(x+iy))$.

4. Evaluate the contour integral $\int_C e^z dz/(z-2)$ if C is the circle of center 0 and radius 3.

5. Find the Laurent series for $\sin(\pi z)/(4z^2-1)$ about the point $z=1/2$ and use this to find the residue at $z=1/2$.

6. Evaluate the integral
$$\int_{-\infty}^{\infty} \frac{\cos(x)}{x^2+1}dx$$
(Hint: $\cos(x)=Re(\exp(ix))$.)

7. Evaluate the integral
$$\int_0^{2\pi} \frac{d\theta}{5-3\cos(\theta)}$$
(Hint: put $z=e^{i\theta}$.)

8. A flat plate is in the shape of a quarter circle of radius 1, and consists of the points (x,y) in the plane with $x\geq 0$, $y\geq 0$, $x^2+y^2\leq 1$. The curved part of the boundary is insulated and the edges $y=0$ and $x=0$ are held at temperatures of 0^o and 100^o. Find the temperature distribution $T(x,y)$ inside the plate. (Hint: if the quarter circle is regarded as part of the unit circle in the z-plane, then the mapping function $w=\log(z)$ maps the quarter circle to an infinitely long rectangle in the w-plane.)