FINAL

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Mathematical Tools for the Physical Sciences
121A
Fall 2002: Egilsson

Friday, Dec. 13, 2002 from 12:30 to 3:30PM in Room 3 Evans Hall

Name:_____.

REMEMBER TO SUPPLY ALL NECESSARY ARGUMENTS

1 (3%) Find the first few nonzero terms of the Maclaurent series of the function,

$$f(x) = \int_0^x \frac{dt}{1 - t^4}.$$

2 (3%) Write the complex number $\frac{2i}{i-1}$ in polar form $re^{i\theta}$. Note $i = \sqrt{-1}$.

3 (3%) Find the radius of convergence for the series,

$$\sum_{n=1}^{+\infty} n^3 (2iz)^n.$$

4 (3%) Write $\cos(\ln(z+1))$ in rectangular form $x + iy$, and show all your calculations as always.

5 (3%) If $y\cos(xy) = \sin(\pi x)$ find $\frac{dy}{dx}$ at $y = 0, \quad x = 0$.

6 (4%) Find one solution f(x,y) to the differential equation,

$$\frac{\partial^2 f}{\partial x^2} - \frac{\partial^2 f}{\partial y^2} = e^{x+y}.$$

Hint: Let $x = u + v$, $y = u - v$ and rewrite the equation in the variables u and v.

7 (3%) Evaluate the path integral,

$$\int_\gamma \text{Re}(z)dz$$

where γ is the unit circle centered at the origin and oriented counterclockwise.

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8 (4%) Evaluate the path integral,

$$\int_{1-i\infty}^{1+i\infty} \frac{z}{z(z^2-4)} dz.$$

9 (3%) Find the residue at $z = \pi$ of the function,

$$f(z) = \frac{ze^z}{(z-\pi)^7}.$$

10 (4%) Calculate, directly, the Laplace integral transform of the function $f(t) = \int_0^t g(\tau)d\tau$ given that the Laplace transform of g is $L(g)(p) = \frac{1}{p}\tanh(p)$.

11 (3%) Find the Laplace transform of $g(t)$ if the function satisfies the equations: $g''(t) - 8g'(t) + 16g(t) = 1$ and $g(0) = 1$, $g'(0) = 2$.

12 (4%) Calculate the Fourier integral transform of the function $f(x) = e^{-|x|} \cos(x)$.