

Fall 2001 Professor Borchers 1st Midterm
Math 121A midterm, Thursday September 27, 9:40-11:00.

Please make sure that your name is on everything you hand in.

You are allowed calculators and 1 page of notes.

Answer as many questions as you can.

All questions have about the same number of marks.

1. Test the series $\sum_{n=1}^{\infty} n/(n^2 + 1)$, $\sum_{n=2}^{\infty} 1/n \log(n)$ and $\sum_{n=1}^{\infty} 100^n/n!$ for convergence.
2. Evaluate $\lim_{x \rightarrow 0} (1 - e^{x^3})/x^3$.
3. Express each of the following complex numbers in the form $a + ib$ for a and b real: $(1 + i)/(2 + i)$, $(1 + i)^{10}$, $e^{3\pi i/2}$.
4. Evaluate $1 + \cos(\theta) + \cos(2\theta) + \cdots + \cos(n\theta)$.
5. If $x^3 + ay = b$ and $y^3 + bx = a$ find $(\partial x/\partial a)_b$, $(\partial x/\partial a)_y$, $(\partial a/\partial x)_b$, $(\partial a/\partial x)_y$ at $(x, y, a, b) = (-1, 2, 3, 5)$.
6. If $xe^y = ye^x$ find dy/dx and d^2y/dx^2 for $y \neq 1$.
7. Find all possible values of i^{1+i} .
8. Find a formula for $\cos(5\theta)$ as a polynomial in $\cos(\theta)$ and $\sin(\theta)$.