

10 Evans
8–11 AM

Your Name: _____

TA: _____

This booklet comprises a cover sheet, eight pages of questions, and a formula sheet at the back. Please check that your booklet is complete and write your name on this cover sheet and the question sheets. Some of the questions have low point values; leave them until the end if you can't do them right away. As usual:

- You need not simplify your answers unless you are specifically asked to do so.
- It is essential to write legibly and *show your work*.
- If your work is absent or illegible, and your answer is not perfectly correct, then no partial credit can be awarded.
- Completely correct answers which are given without justification may receive little or no credit.
- During this exam, you are not allowed to use calculators or consult your notes or books.

Problem	Maximum	Your Score
1	8	
2	8	
3	9	
4	7	
5	19	
6	8	
7	9	
Total	68	

At the conclusion of the exam, hand in this exam paper to your TA.

Your Name: _____

1a (3 points). Express as a definite integral the length of the curve $y = \sin x$, $0 \leq x \leq \pi/2$.

1b (5 points). Decide whether $\sum_{n=1}^{\infty} (-1)^n \frac{2^n n!}{5 \cdot 8 \cdot 11 \cdots (3n+2)}$ converges absolutely, converges conditionally, or diverges.

Your Name: _____

2a (4 points). Find $f^{(666)}(0)$ if $f(x) = \sec(x^{333})$.

2b (4 points). A tank contains 500 L of pure water. Brine that contains 0.05 kg of salt per liter flows into the tank at the rate of 8 L/min. The solution is kept thoroughly mixed and drains from the tank at the rate of 8 L/min. Let $y(t)$ be the amount of salt in the solution at time t , measured in kg; time is measured in minutes. What differential equation is satisfied by y ? Without solving this equation, guess the value of $\lim_{t \rightarrow \infty} y(t)$.

Your Name: _____

3a (4 points). Find one solution to $y'' + 5y' + 6y = \cos x$.

3b (5 points). Use power series methods to solve the initial-value problem:
 $y'' + xy' - 2y = 0$, $y(0) = 1$, $y'(0) = 0$.

Your Name: _____

4a (3 points). Let $y = f(x)$ be the solution to the initial-value problem $y' = y^2 + 2y$, $y(2) = 1$. Without finding a formula for $f(x)$, compute $f'(2)$ and $f''(2)$.

4b (4 points). Find the equation of the curve which passes through $(1, 1)$ and which is orthogonal to the family of curves $2x^2 + y^2 = k$.

Your Name: _____

Evaluate each of the following integrals whenever possible (beware of divergent improper integrals):

5a (5 points). $\int_0^2 \sqrt{2x - x^2} dx.$

5b (5 points). $\int_{-5}^5 \frac{4 dt}{t^2 - 2t - 3}.$

Your Name: _____

5c (5 points). $\int \cos \sqrt{x} dx.$

5d (4 points). $\int_e^\infty \frac{dx}{x(\ln x)^2}.$

Your Name: _____

6a (4 points). Test for convergence: $\sum_{n=0}^{\infty} \sqrt{\frac{2^n + 3^n}{6^n}}$.

6b (4 points). How many terms of the alternating series $\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!}$ are required to approximate the actual sum with an error of less than 5×10^{-4} ?

Your Name: _____

7a (4 points). Using a series representation for $(1+x)^{1/2}$, evaluate

$$\sum_{n=2}^{\infty} (-1)^n \frac{(1 \cdot 3 \cdot 5 \cdots (2n-3)) 9^n}{n!(32)^n}.$$

7b (5 points). Solve the differential equation $xy' = 2\sqrt{xy} - y$.