NAME ______________________________

TA's name or section number ______________________________

MATH 1B First Midterm Fall 2001

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There are 200 points altogether. The first 9 questions are multiple choice, worth 15 points each. Choose the most correct answer to each question and mark the corresponding box in the grid below. Mark only one box per question. No partial credit.

Question	a	b	c	d	e
1					
2					
3					
4					
5					
6					
7					
8					
9					

TA use only:

MC	
1	
2	
3	
Total	

Multiple choice questions:

1) Which of the following is correct?
(a) The method of partial fractions shows that the integral of any rational function is a rational function.
(b)The integral of any rational function in $\sec x$ and $\tan x$ is a rational function in $\sin x$ and $\cos x$.
(c)Any function $f(x)$ that is continuous on an interval $[a, b]$ has an antiderivative on that interval.
(d) The integral of any elementary function is an elementary function.
(e) Cricket is the most popular sport in the United States.

2) Which of the following integrals gives the area of the surface obtained by rotating the curve $y = e^x$ for y between $1/e$ and e about the line $x = 2$?

(a)$2\pi \int_{-1}^{1} (2 - e^x)\sqrt{1 + e^{2x}}\, dx$

(b)$2\pi \int_{1/e}^{e} (e^x - 2)\sqrt{1 + e^{2x}}\, dx$

(c)$2\pi \int_{1/e}^{1} (2 - ln(x))\sqrt{1 + e^{2y}}\, dy$

(d)$2\pi \int_{1/e}^{e} (2 - ln(y))\sqrt{1 + \frac{1}{y^2}}\, dy$

(e)$2\pi \int_{-1}^{1} (x - 2)\sqrt{1 + e^{2x}}\, dx$

3) Let $\{a_n\}$ be a decreasing sequence of numbers, bounded below by 4. Which of the following is *incorrect*?
(a) The sequence $\{a_n\}$ does not necessarily have a limit as $n \to \infty$.
(b) The sequence $\{a_n\}$ must have a limit as $n \to \infty$ and that limit is ≥ 4.
(c) The sequence $\{a_n - a_{n+1}\}$ tends to 0 as $n \to \infty$.
(d) The sequence $\{a_n + a_{n+1}\}$ has a limit as $n \to \infty$ and that limit is ≥ 8.
(e) The sequence $\{a_n^2\}$ has a limit as $n \to \infty$ and that limit is ≥ 16.

4) To integrate the function $\frac{x^3}{x^3 - 1}$ by partial fractions one should try to express it in the form

(a) $1 + \frac{A}{x-1} + \frac{Bx + C}{x^2 + x + 1}$

(b)$\frac{A}{x^3} - \frac{B}{x^2} + \frac{C}{x}$

(c)$\frac{A}{x^3} + \frac{B}{x^2} - \frac{C}{x}$

(d)$\frac{A}{x-1} + \frac{Bx + C}{x^2 + x + 1}$

(e)$\frac{A}{x-1} - \frac{Bx + C}{x^2 + x + 1}$

5)The length of the curve $y = e^{|x|}$ for $-1 \leq x \leq +1$ is

(a) $\int_{-1}^{1} \sqrt{1 - e^{2|x|}}\, dx$

(b) $2\int_{0}^{1} \sqrt{1 + e^{2x}}\, dx$

(c) $2\int_{-1}^{1} \sqrt{1 - e^{2x}}\, dx$

(d) $\int_{-1}^{1} \sqrt{1 - e^{|x|}}\, dx$

(e) $\int_{-1}^{1} \sqrt{1 + e^{|x|}}\, dx$

6)Which of the following is always true for a convergent series $\sum_{i=1}^{\infty} a_i$?

(a)The partial sums $\sum_{i=1}^{n} a_n$ tend to 0 as $n \to \infty$.

(b) $\lim_{i\to\infty} (a_i + a_{i+1}) = 0$

(c) The sequence of absolute values $\{|a_i|\}$ is decreasing.

(d) The sequence of absolute values $\{|a_i|\}$ is increasing.

(e) The numbers a_i all have the same sign.

7)Which of the following is true for any sequence $\{a_n\}$ with $\lim_{n\to\infty} a_n = 4$?

(a) There is an $N > 0$ for which $a_n < 2$ for all $n \leq N$.

(b) There is an N for which $|a_n - 4| < 1$ for all $n \geq N$.

(c) $\lim_{n\to\infty}(a_n + a_{n+1}) = \infty$.

(d) For no value of n is a_n bigger than 300.

(e)For any $\epsilon > 0$ there is an N with $|a_n - 4| > \epsilon$ for all $n \geq N$.

8)The indefinite integral $\int x^2 e^{x^2}\, dx$ is

(a) $\frac{e^{x^2}}{2} + C$

(b) $\frac{xe^{x^2}}{2} + C$

(c) $\frac{e^{-x^2}}{2} + C$

(d) $\frac{x^3}{3}e^{x^2} + 2x^3e^{x^2} + C$

(e) impossible to do in terms of elementary functions.

9) Which of the following is correct concerning integration by parts?

(a) $\int f(x)g(x)dx = f(x)\int g(x)dx + \left(\int f(x)dx\right)g(x)$
(b) $\int f(x)g(x)dx = f(x)\int g(x)dx + \left(\int f(x)dx\right)g(x) + C$
(c) It gives a formula for integrating the product of two functions $f(x)$ and $g(x)$ in terms of $\int f(x)dx$ and $\int g(x)dx$.
(d) It is the integration formula derived from the product rule for differentiation.
(e)It can only be used on a product $f(x)g(x)$ if the integrals of both f and g are known already.

The next three questions are *not* multiple choice. Show your reasoning and give your answers in the space provided.
NOTE THAT THERE ARE QUESTIONS ON BOTH SIDES OF THE PAGE!!!!!
1.
a) Evaluate the following integrals:(each worth 18 points)

(i) $\int_0^4 x^2\sqrt{16-x^2}dx$

(ii) $\int \frac{x}{x^2 - x + 6} dx$

b)(15 points) Evaluate $\int_0^\infty \frac{dx}{x^2 - 5}$ or show that it is divergent.

2.(20 points) The error in estimating $\int_a^b f(x)\,dx$ using Simpson's rule with n intervals is at most $\frac{K(b-a)^5}{180n^4}$ when $|f^{(4)}(x)| \le K$ for $a \le x \le b$.

How large should n be to be sure the error is less than 10^{-5} in estimating

$\int_1^3 \frac{3\,ln(x)}{2}$ using Simpson's rule?

3. (a)(20 points) Is the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 3n + 2}$ convergent? If so find its sum.

(b)(10 points) Suppose f is a continuous positive strictly decreasing function for $x \geq 1$ and $a_n = f(n)$. By drawing a picture, rank the following in increasing order:

$$\int_1^6 f(x)\,dx \qquad \sum_{i=1}^{5} a_i \qquad \sum_{i=2}^{6} a_i.$$