SOLUTIONS

ME 105 Midterm #2

11/13/06

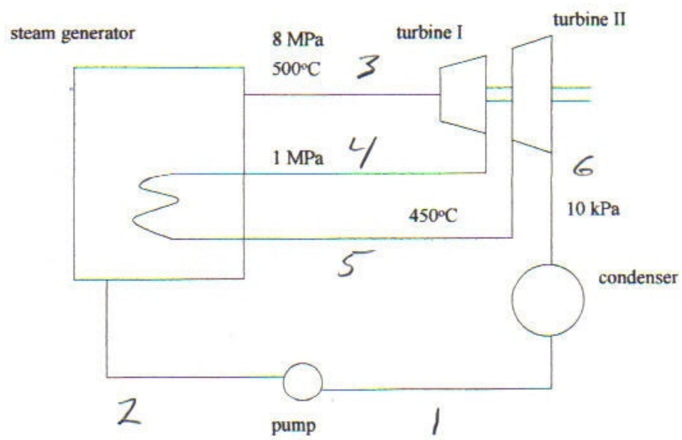
Problem 1 (50%)

Steam is the working medium in an ideal Rankine cycle with reheat (see figure below). Steam enters the first stage turbine at 8 MPa, 500°C and expands to 1 MPa. It is then reheated to 450°C before entering the second-stage turbine where it expands to the condenser pressure of 10 kPa. The water then leaves the condenser as saturated liquid. The net power output is 100 MW. Please determine:

(a) the efficiency of the cycle

(b) the mass flow rate of steam

(c) the rate of heat transfer from the condensing steam as it passes through the condenser.



$h_1 = h_{f@10kPa} = 191.81\ kJ/kg$

$v_1 = v_{f@10kPa} = 0.001010\ m^3/kg$

$h_2 = h_1 + v_1(P_2 - P_1) = 199.88\ kJ/kg$

$h_3 = h(8MPa, 500°C) = 3399.5\ kJ/kg$

$s_3 = s(8MPa, 500°C) = 6.7266\ kJ/kg\cdot K$

$s_4 = s_3 = 6.7266\ kJ/kg\cdot K$

$$h_4 = h(1MPa, 200°C) + \frac{s_4 - s(1MPa, 200°C)}{s(1MPa, 250°C) - s(1MPa, 200°C)}\left[h(1MPa, 250°C) - h(1MPa, 200°C)\right]$$

$= 2843.7\ kJ/kg$

$h_5 = h(1 MPa, 450°C) = 3371.8 kJ/kg$

$s_5 = s(1 MPa, 450°C) = 7.6156 kJ/kg \cdot K$

$s_6 = s_5 = 7.6156 kJ/kg \cdot K$

$$x_6 = \frac{s_6 - s_{f@10kPa}}{s_{fg@10kPa}} = 0.9289$$

$h_6 = h_{f@10kPa} + x_6 h_{fg@10kPa} = 2413.8 kJ/kg$

(a)

$q_{in} = (h_5 - h_4) + (h_3 - h_2) = 3727.7 kJ/kg$

$w_{net,out} = w_{T1} + w_{T2} - w_P = (h_3 - h_4) + (h_5 - h_6) - (h_2 - h_1)$

$= 1505.7 kJ/kg$

$$\eta_{th} = \frac{w_{net,out}}{q_{in}} = \boxed{0.404}$$

(b)

$$\dot{m} = \frac{\dot{W}_{net,out}}{w_{net,out}} = \frac{100 \times 10^3 kJ/s}{1505.7 kJ/kg} = \boxed{66.4 kg/s}$$

(c)

$q_{out} = q_{in} - w_{net,out} = h_6 - h_1 = 2222.0 kJ/kg$

$\dot{Q}_{out} = \dot{m} q_{out} = \boxed{147.5 MW}$

Problem 2 (50%)

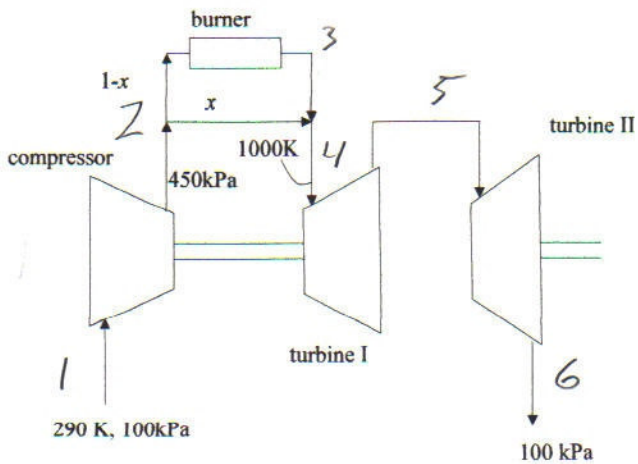
A gas turbine has two ideal sections as shown in the figure below. Section I drives the ideal compressor and section II produces the power output. The compressor input is at 290K, 100 kPa, and the exit is at 450 kPa. A fraction of the flow *x* bypasses the burner and the rest 1-*x* fraction goes through the burner where heat of 1000 kJ/kg is added by combustion. The two flows then mix before entering the first turbine and continue through the second turbine with the exhaust at 100 kPa. The temperature right after mixing and at the inlet of the first turbine is 1000 K. Please determine:

(a) mixing fraction, *x*.
(b) the required pressure into the second stage of the turbine
(c) the net power output per unit mass flow rate

You may assume that the burner is modeled as a simple heat addition and that the working medium is air throughout with constant specific heats at ambient temperature.

$C_p = 1.005\ kJ/kg\cdot K$

$k = 1.4$



(a)

$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k}$ $\quad T_2 = 445.7K$

$q_{in} = h_3 - h_2 = C_p(T_3 - T_2) = 1000\ kJ/k \Rightarrow T_3 = 1440.7K$

$x h_2 + (1-x) h_3 = h_4 \Rightarrow x = \frac{h_4 - h_3}{h_2 - h_3} = \frac{C_p(T_4 - T_3)}{C_p(T_2 - T_3)} \Rightarrow \boxed{x = 0.443}$

(b) $W_{c,in} = W_{T_1,out} \Rightarrow C_p(T_2 - T_1) = C_p(T_4 - T_5) \Rightarrow T_5 = 844.3K$

$\frac{T_5}{T_4} = \left(\frac{P_5}{P_4}\right)^{(k-1)/k} \Rightarrow P_5 = P_4\left(\frac{T_5}{T_4}\right)^{k/(k-1)} \Rightarrow \boxed{P_5 = 248.9 kPa}$

(c) $\left(\frac{T_6}{T_5}\right) = \left(\frac{P_6}{P_5}\right)^{(k-1)/k} \Rightarrow T_6 = 650.6K$

$W_{net,out} = h_5 - h_6 = C_p(T_5 - T_6)$ $\quad \boxed{W_{net} = 194.7\ kJ/kg}$