Name _____

TA _____

Math 1B — Second Midterm

V.Jones, Spring 1998

150 points total. The first 5 questions are Multiple Choice.
For each question mark an $\times$ in the most correct place
in the grid below. No partial credit for 1–5.
Questions 6, 7 and 8 are not multiple choice.

	a	b	c	d	e
1					
2					
3					
4					
5					

MC _____

6 _____

7 _____

8 _____

Math 1B Midterm

Multiple Choice Questions. Each multiple choice question worth 15 points.

1. Consider the series $\sum_{n=1}^{\infty} \frac{\cos\left(\frac{n\pi}{3}\right)}{n}$. Which of the following is correct?

   a) The series converges absolutely.

   b) The series converges but not absolutely.

   c) The series does not converge because the n^{th} term does not tend to zero.

   d) The series does not converge, by the ratio test.

   e) The series does not converge, by comparison with $\sum_{n=1}^{\infty} \frac{1}{n}$.

2. Suppose numbers a_n, for $n \geq 0$, are given such that $\sum_{n=0}^{\infty} 2^n a_n$ converges. Which of the following is a consequence?

a) $\lim_{n\to\infty} a_n = 0$

b) $\lim_{n\to\infty} a_n = 1$

c) $\sum_{n=0}^{\infty} \frac{a_n}{2^n}$ converges

d) $\sum_{n=0}^{\infty} a_n$ diverges

e) $\lim_{k\to\infty} \left(\sum_{n=0}^{k} 2^n a_n \right) = 0$

3. The MacLaurin series for $\sin^2 x$ is

a) $x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

b) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

c) $x + \frac{x^5}{5!} + \frac{x^9}{9!} + \frac{x^{13}}{13!} + \dots$

d) $1 - x + \frac{x^3}{3!} - \frac{x^5}{5!} + \dots$

e) $\frac{1}{2}\left\{ \frac{4x^2}{2!} - \frac{16x^4}{4!} + \frac{64x^6}{6!} - \frac{256x^8}{8!} + \dots \right\}$

4. Which of the following is an example of a sequence with $\lim_{n\to\infty} a_n = -\infty$?

a) $a_n = (-1)^n n$

b) $a_n = (-1)^{n^2} n^2$

c) $a_n = \frac{1}{n}(e^n - e^{n^2})$

d) $a_n = \dfrac{n \ln(n+5)}{(n+1)(n+5)}$

e) $a_n = \cos n.$

5. Let $J_0(x)$ be the Bessel function $\sum_{n=0}^{\infty} \dfrac{(-1)^n x^{2n}}{2^{2n}(n!)^2}$. Which of the following is correct?

a) $x^2 \dfrac{d^2 J_0}{dx^2} + x \dfrac{dJ_0}{dx} + x^2 J_0 = 0$

b) $J_0(x) = e^x \cos(\ln x).$

c) $\lim_{x\to 0} J_0(x) = 2$

d) $\int_0^t J_0(x)dx = J_0(t)$

e) $\dfrac{dJ_0}{dx} = J_0$

Longer Questions

6. *(15 pts)* (a) Find the MacLaurin series for $\dfrac{x^2}{(1-x)^2}$.

(15 pts) (b) Find the sum of the series $\displaystyle\sum_{n=2}^{\infty} n\left(\frac{1}{2}\right)^n$.

7. *(15 pts)* Consider the series $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} \ldots$. What is the last term of the series that would have to be included to obtain $\pi/4$ to within 0.01. Justify your answer.

8. For each of the following series, say whether it converges absolutely, conditionally, or diverges. Give reasons.

(15 pts) (i) $\sum_{n=0}^{\infty} (-1)^n \frac{n}{n+50}$

(15 pts) (ii) $\sum_{n=1}^{\infty} \frac{\ln(n)}{n^2}$