# MATH 1B SECOND MIDTERM

**October 25, 2002**

**2050 VLSB H. Wu**

**Your Name:** _____

**Your GSI's Name:** _____

**Instructions**

(1). Check that you have all **6** pages of this exam booklet.

(2). *Be sure to show all your steps.*

(3). **You may not use any fact that has not been covered in the course to do the exam.**

| EXAM SCORES | | | | | |
|---|---|---|---|---|---|
| | Max | Your score | | Max | Your score |
| I | 30 | | VI | 20 | |
| II | 10 | | V | 20 | |
| III | 20 | | | | |
| TOTAL | | | % | | |

**Your Name:** _____

**I.** (30%) Determine whether the given series is convergent or divergent. You may assume as known the fact that $\sum_n (1/n^p)$ converges if $p > 1$ and diverges if $p \leq 1$, and that $\sum_n r^n$ converges if $|r| < 1$ and diverges if $|r| \geq 1$.

(a) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2 - \sin n}{4}\right)^n$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{(n+5)^3}$

(c) $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^{\ln n}}$

**II.** (10%) Find the sum of the series: $\sum_{n=1}^{\infty} \left( \frac{7}{10^{2n-1}} + \frac{1}{10^n} \right) = \frac{7}{10} + \frac{1}{10^2} + \frac{7}{10^3} + \frac{1}{10^4} + \frac{7}{10^5} + \frac{1}{10^6} + \cdots$

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**III.** (20%) Determine the radius of convergence and the interval of convergence of the following power series: $\sum_{n=3}^{\infty} \frac{x^n}{\ln n}$ Give reasons.

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**IV.** (20%) Given a sequence $\{a_n\}$ so that $a_1 = 1$ and, for any integer $n \geq 1$, $a_{n+1} = \frac{1}{5}(a_n + 7)$.

(a) Show that $a_n < a_{n+1} < 2$ for all $n$, and (b) explain why $\{a_n\}$ is convergent and find $\lim_{n\to\infty} a_n$.

V. (20%) Give a direct explanation, *without* making use of the Integral Test, of why $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ is divergent. (Note that the sum starts with $n = 2$.) You may of course use the *reasoning* that proves the Integral Test.