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Fall 1999, Math 1B

Second Midterm

26 October, 1999
8:10-9:30 AM

1. (36 points, 6 points apiece) Find the following. If an expression is undefined, say so.

(a) $\Sigma_{n=2}^{\infty} 5^{-n}$.

(b) $\Sigma_{n=1}^{\infty} (2^n + 2^{-n})$.

(c) The set of all real numbers p such that $\Sigma_{n=2}^{\infty} n^{-1} (\ln n)^p$ converges.

(d) The Maclaurin series for 2^x.

(e) The Taylor series for $1/x^2$ centered at $x = 1$.

(f) The solution to the differential equation $xy' = (x+1)\,y$ satisfying the initial condition $y(1) = 1$.

2. (16 points) Let a and b be real numbers. Prove that $\Sigma_{n=1}^{\infty} (\frac{a}{n} + \frac{b}{n+1})$ converges if and only if $a + b = 0$.

3. (30 points, 6 points apiece) For each of the items listed below, give either *an example*, or a brief reason why *no example exists*. (If you give an example, you are *not* asked to show that it has the asserted property.)

(a) A power series $\Sigma_{n=0}^{\infty} a_n (x-1)^n$ with radius of convergence 3.

(b) A power series $\Sigma_{n=0}^{\infty} a_n x^n$ which converges for all $x \geq -1$ and no other x.

(c) A power series $\Sigma_{n=0}^{\infty} a_n (x-2)^n$ which converges for all real numbers x.

(d) A series $\Sigma_{n=1}^{\infty} a_n$ which is convergent but not absolutely convergent.

(e) Two series $\Sigma_{n=1}^{\infty} a_n$ and $\Sigma_{n=1}^{\infty} b_n$ such that $a_n \geq b_n$ for all n, and $\Sigma_{n=1}^{\infty} a_n$ converges, but $\Sigma_{n=1}^{\infty} b_n$ diverges.

4. (18 points) (a) (7 points) Find the first three terms (i.e., the constant, linear, and square terms) of the Taylor series for $\ln x$ centered at $x = 2$.

(b) (11 points) Prove using the formula for the remainder (''Taylor's Formula'') that for all x in the interval $[1.5, 2.5]$, the sum of the above three terms approximates $\ln x$ to within $1/81$.