Math 1B Spring, 1994 Professor K. Ribet

First Midterm Exam — February 22, 1994

1a *(6 points)*. Calculate $\int \frac{2u\,du}{u^2 - 2u + 5}$.

1b *(3 points)*. Write a definite integral which represents the length of the curve $y = \tan x$, $0 \le x \le \pi/4$.

2 *(7 points)*. Find the area of the surface obtained by rotating the curve $y = \sin x$, $0 \le x \le 2\pi$ about the line $y = 0$.

Decide whether each of the following sequences converge or diverge. In the case of a convergent sequence, find the limit. Explain your reasoning!

3a *(4 points)*. $a_n = \frac{3^n}{\pi^n}$;

3b *(4 points)*. $b_n = \begin{cases} n\sin(1/n) & \text{if } n \text{ is odd} \\ (1+n)^{1/n} & \text{if } n \text{ is even;} \end{cases}$

3c *(4 points)*. $c_n = \frac{1}{n^2} + \frac{2}{n^2} \cdots + \frac{n}{n^2}$.

4a *(7 points)*. Evaluate $\int_0^1 x \sin^{-1} x\,dx$.

4b *(7 points)*. Suppose that $f(0) = 3$. Use Simpson's rule with $n = 4$ to estimate $f(8)$:

x	0	2	4	6	8
$f'(x)$	2	4	1	3	5

5 *(7 points)*. Write as a sum of partial fractions: $\frac{x^3 + 5x^2 + 2x + 4}{x^4 + 2x^2}$.

6a *(6 points)*. Evaluate the improper integral $\int_1^\infty \left(\frac{2x}{x^2+1} - \frac{2}{x+1}\right) dx$. (If the integral is divergent, answer "divergent" and explain your reasoning.)

6b *(5 points)*. Find $\int_{\ln 2}^{\ln 3} \frac{dx}{e^x - 1}$.

Second midterm exam — April 7, 1994

1a *(4 points)*. If the series $\sum_{n=1}^{\infty} (1 - \sqrt{2})^n$ converges, calculate its value. If it is divergent, explain why.

1b *(5 points).* Use Taylor series to express $\int_0^1 \frac{\sin x}{x}\,dx$ as an infinite series.

2a *(5 points).* Find the radius of convergence and the interval of convergence of the infinite series $\sum_{n=1}^{\infty} \frac{(x-2)^{2n}}{n^3}$.

2b *(4 points).* For what positive values of x does the series $\sum_{n=1}^{\infty}(x+\frac{x}{n})^n$ converge?

3 *(7 points).* Show that $|\sin x - x + \frac{x^3}{6}| < 10^{-7}$ if $0 < x < \frac{1}{10}$. [Consider the Maclaurin polynomial of degree 4 for $\sin x$.]

4a *(6 points).* Find $y(t)$, given that $ty' + y = -\sin t$ and that $y(\pi/2) = 0$.

4b *(6 points).* Solve the homogeneous differential equation $y' = \frac{x-y}{x}$.

Decide whether each of the following series converges or diverges. Explain your reasoning!

5a *(4 points).* $\sum_{n=1}^{\infty}(-1)^n n \sin\frac{1}{n}$;

5b *(4 points).* $\sum_{n=4}^{\infty}\frac{n!}{n^n}$;

5c *(4 points).* $\sum_{n=1}^{\infty}\frac{(\ln n)^{10!}}{n\sqrt{n}}$;

5d *(4 points).* $\frac{1}{2}+\frac{1}{2\cdot 3}+\frac{1}{2\cdot 2\cdot 3}+\frac{1}{2\cdot 2\cdot 3\cdot 3}+\frac{1}{2\cdot 2\cdot 2\cdot 3\cdot 3}+\frac{1}{2\cdot 2\cdot 2\cdot 3\cdot 3\cdot 3}+\cdots$.

6 *(7 points).* Find the Maclaurin series for $f(x) = \frac{1}{\sqrt[3]{8-x}}$. What is the radius of convergence of this series?

Final exam — May 21, 1994

1b *(6 points).* The alternating series $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/n}}$:

- **a.** Converges conditionally; ____
- **b.** Converges absolutely; ____
- **c.** Diverges; ____

1c *(7 points).* Evaluate the improper integral $\int_{-1}^{+1}\sqrt{\frac{1-x}{1+x}}\,dx$. (If the integral is divergent, answer "divergent" and explain your reasoning.)

2a *(6 points)*. For which values of x does the series $\sum_{n=0}^{\infty}\left(x^n + (3x^2)^{n+1}\right)$ converge? Find the sum of the series for those values.

2b *(7 points)*. Consider the differential equation $\frac{dN}{dt} = (N+1)(N-2)(N-3)(N-4)$. Suppose that $N(t)$ is a solution to the equation with $N(0) = 1.5$. Then $\lim_{t\to\infty} N(t)$:

a. Is necessarily -1 ____;

b. Is necessarily 2 ____;

c. Is necessarily a number other than -1 or 2 ____;

d. Can be any real number ____;

e. Does not exist ____.

3a *(6 points)*. Find $\int (\cos x + \sin x)^2 \tan x \, dx$.

3b *(5 points)*. Given $f(0) = 0$, use Simpson's rule with $n = 4$ to estimate $f(8)$:

x	0	2	4	6	8
$f'(x)$	1	2	3	5	4

4 *(9 points)*. Calculate the integral $\int \frac{x^2+x+1}{(x+1)(x^2+1)}\,dx$. Check your work carefully!

5a *(6 points)*. Find $\lim_{n\to\infty} \sqrt[n]{2^n + 4^n}$.

5b *(6 points)*. Find the equation of the curve which passes through $(1,1)$ and whose slope at each point (x, y) on the curve is y^2/x^3.

6a *(7 points)*. Find $f^{(19)}(0)$ where $f(x) = (1+x)\cos x$.

6b *(6 points)*. Determine whether the sum $\sum_{n=1}^{\infty} \frac{\sqrt{n+1}-\sqrt{n}}{n}$ converges.

7 *(8 points)*. A tank contains 25 kg of salt dissolved in 500 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at the rate of 25 L/hour. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the mixture after t hours?

8a *(6 points)*. Find $f(x)$ given: $f''(x) - 2f'(x) + 2f(x) = 0$, $f(0) = 1$, $f'(0) = 1$.

8b *(7 points)*. Solve the differential equation $y'' + y = x + \cos x$.

9 *(8 points)*. Find a series solution for $y'' = 2xy' + 2y$ with the initial conditions $y(0) = 1$, $y'(0) = 0$.