MATHEMATICS 1B
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Midterm #1 Fall 2003

Instructions Write you name , GSI's name , and your section number on your Blue Book right now! Show all your work. Partial credit may be given if the work justifies it. Best wishes on the exam!

Problem #1 Find the following integrals.

$$(A) \quad \int (x+3)e^x dx \qquad\qquad (B) \quad \int tan^2(x) dx$$

Problem #2 Find the following integrals.

$$(A) \quad \int cos(3\sqrt{x}) dx \qquad\qquad (B) \quad \int \frac{1}{\sqrt{x^2-8x+5}} dx$$

Problem #3 Write out the partial fractions expansion for

$$\frac{3}{x^3(x^2+7)}$$

Do not solve for the coefficients on the right hand side.

Problem #4 Consider the curve defined by the two following equivalent descriptions: (A) $y = sin(x)$ for $0 \le x \le \pi/2$ and (B) $x = sin^{-1}(y)$ for $0 \le y \le 1$. In each case, write down the specific formula for the area S of the surface obtained by revolving this curve about the x-axis. You do not need to compute the value of S.

Problem #5 Which number in the list {35,40,55,70 } is the least integer n so that the midpoint rule approximation M_n to the definite integral

$$\int_0^1 x^3 - x \, dx$$

will be accurate to 4 decimal places?

Problem #6 Compute

$$lim_{n\to\infty} cos(\frac{\pi}{n}) \frac{\sqrt{3n^2+n}}{\sqrt{n^2-n+1}}$$

You can assume that

$$\lim_{n\to\infty} \frac{\alpha}{n^p} = 0$$

where p is an integer greater than or equal to 1 and α is any constant.

Problem #7 Determine whether the integral

$$\int_1^\infty \frac{sin^2(x)}{\sqrt{x+x^7}} dx$$

is convergent or divergent.

Tables on back $\Rightarrow$

Table of Integrals and Trig Identities

Basic Forms

1. $\int u\,dv = uv - \int v\,du$
2. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C, \quad n \neq -1$
3. $\int \dfrac{du}{u} = \ln|u| + C$
4. $\int e^u\,du = e^u + C$
5. $\int a^u\,du = \dfrac{a^u}{\ln a} + C$
6. $\int \sin u\,du = -\cos u + C$
7. $\int \cos u\,du = \sin u + C$
8. $\int \sec^2 u\,du = \tan u + C$
9. $\int \csc^2 u\,du = -\cot u + C$
10. $\int \sec u \tan u\,du = \sec u + C$
11. $\int \csc u \cot u\,du = -\csc u + C$
12. $\int \tan u\,du = \ln|\sec u| + C$
13. $\int \cot u\,du = \ln|\sin u| + C$
14. $\int \sec u\,du = \ln|\sec u + \tan u| + C$
15. $\int \csc u\,du = \ln|\csc u - \cot u| + C$
16. $\int \dfrac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\dfrac{u}{a} + C$
17. $\int \dfrac{du}{a^2 + u^2} = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$
18. $\int \dfrac{du}{u\sqrt{u^2 - a^2}} = \dfrac{1}{a}\sec^{-1}\dfrac{u}{a} + C$
19. $\int \dfrac{du}{a^2 - u^2} = \dfrac{1}{2a}\ln\left|\dfrac{u+a}{u-a}\right| + C$
20. $\int \dfrac{du}{u^2 - a^2} = \dfrac{1}{2a}\ln\left|\dfrac{u-a}{u+a}\right| + C$

Forms involving $\sqrt{a^2 + u^2}$, $a > 0$

21. $\int \sqrt{a^2 + u^2}\,du = \dfrac{u}{2}\sqrt{a^2 + u^2} + \dfrac{a^2}{2}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
22. $\int u^2\sqrt{a^2 + u^2}\,du = \dfrac{u}{8}(a^2 + 2u^2)\sqrt{a^2 + u^2} - \dfrac{a^4}{8}\ln\left(u + \sqrt{a^2 + u^2}\right) + C$
23. $\int \dfrac{\sqrt{a^2 + u^2}}{u}\,du = \sqrt{a^2 + u^2} - a\ln\left|\dfrac{a + \sqrt{a^2 + u^2}}{u}\right| + C$
24. $\int \dfrac{\sqrt{a^2 + u^2}}{u^2}\,du = -\dfrac{\sqrt{a^2 + u^2}}{u} + \ln\left(u + \sqrt{a^2 + u^2}\right) + C$
25. $\int \dfrac{du}{\sqrt{a^2 + u^2}} = \ln\left(u + \sqrt{a^2 + u^2}\right) + C$

$\csc\theta = \dfrac{1}{\sin\theta} \qquad \sec\theta = \dfrac{1}{\cos\theta} \qquad \cot\theta = \dfrac{1}{\tan\theta}$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} \qquad \cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sin(x + y) = \sin x\cos y + \cos x\sin y$

$\cos(x + y) = \cos x\cos y - \sin x\sin y$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\cos^2 x = \dfrac{1 + \cos 2x}{2}$

$\sin^2 x = \dfrac{1 - \cos 2x}{2}$

Tables on back $\Longrightarrow$

#1

A) $\int (x+3)e^x\,dx$ $\quad u = x+3 \quad du = dx$
$\quad dv = e^x dx \quad v = e^x$ $\quad$ 4/4

$$\int (x+3)e^x\,dx = (x+3)e^x - \int e^x\,dx$$
$$= (x+3)e^x - e^x + C$$
$$= xe^x + 3e^x - e^x + C$$
$$= \boxed{xe^x + 2e^x + C}$$

B) $\int \tan^2(x)\,dx = \int (\sec^2 x - 1)\,dx$

$$= \int \sec^2 x\,dx - \int dx$$ $\quad$ 3/3

$$= \boxed{\tan x - x + C}$$

#2

A) $\int \cos(3\sqrt{x})\,dx$ $\quad u = 3\sqrt{x} = 3x^{1/2}$

$du = \frac{3}{2}x^{-1/2}\,dx$

$dx = \frac{2}{3}x^{1/2}\,du$

$= \frac{2}{9}u\,du$

$$\int \cos(3\sqrt{x})\,dx = \frac{2}{9}\int u\cos u\,du$$

$w = u \quad dw = du$

$dv = \cos u\,du \quad v = \sin u$

$$\frac{2}{9}\int u\cos u\,du = \frac{2}{9}\left(u\sin u - \int \sin u\,du\right) = u\sin u + \cos u$$

$$\frac{2}{9}\left(3\sqrt{x}\sin(3\sqrt{x}) + \cos(3\sqrt{x})\right) + C$$

Answer: $\frac{2}{3}\sqrt{x}\sin(3\sqrt{x}) + \frac{2}{9}\cos(3\sqrt{x}) + C$

B) $\int \frac{1}{\sqrt{x^2-8x+5}}\,dx$

completing the square

x^2-8x+5

$x^2-8x+16+5-16$

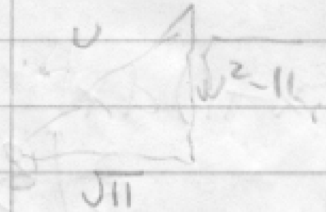
$(x-4)^2-11$

$\int \frac{1}{\sqrt{x^2-8x+5}}\,dx = \int \frac{1}{\sqrt{(x-4)^2-11}}\,dx$

$u = x-4$

$du = dx$

$= \int \frac{1}{\sqrt{u^2-11}}\,dx$



$= \int \frac{1}{\sqrt{11\sec^2\theta-11}}\,dx$

$\cos\theta = \frac{\sqrt{11}}{u}$

$\sec\theta = \frac{u}{\sqrt{11}}$

$\sqrt{11}\sec\theta = u$

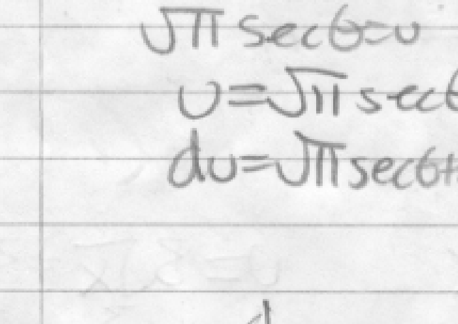
$u = \sqrt{11}\sec\theta$

$du = \sqrt{11}\sec\theta\tan\theta$

$= \int \frac{\sqrt{11}\sec\theta\tan\theta}{\sqrt{11}\sqrt{\sec^2\theta-1}}\,d\theta$

$= \frac{\sqrt{11}}{1}\int \frac{\sec\theta\tan\theta\,d\theta}{\sqrt{\tan^2\theta}}$

$= \frac{\sqrt{11}}{11}\int \frac{\sec\theta\tan\theta\,d\theta}{\tan\theta}$



$= \frac{\sqrt{11}}{11}\int \sec\theta$

$= \frac{\sqrt{11}}{11}\ln|\sec\theta+\tan\theta|+C$

$= \frac{\sqrt{11}}{11}\ln\left|\frac{u}{\sqrt{11}}+\frac{\sqrt{u^2-11}}{\sqrt{11}}\right|+C$

answer: $\ln|x-4+\sqrt{x^2-8x+5}| - \ln\sqrt{11}+C$

$= \frac{\sqrt{11}}{11} \ln \left| \frac{x-4}{\sqrt{11}} + \sqrt{x^2-8x+5} \right| + C$

$= \frac{\sqrt{11}}{11} \ln |x-4+\sqrt{x^2-8x+5}| - \ln\sqrt{11} + C$

$\ln |x-4+\sqrt{x^2-8x+5}| - \ln\sqrt{11} + C$

$\boxed{\#3}$

$$\frac{3}{x^3(x^2+7)} = \frac{A}{x^3} + \frac{B}{x^2} + \frac{C}{x} + \frac{Dx+E}{x^2+7}$$

$\boxed{\#4}$

about x-axis

$y = \sin(x) \quad \frac{dy}{dx} = \cos x$

A) $S = \int_0^{\pi/2} 2\pi \sin(x) \sqrt{1 + (\cos x)^2}\, dx$

$\checkmark = 2\pi \int_0^{\pi/2} \sin x \sqrt{1+\cos^2 x}\, dx$

$x = \sin^{-1} y \quad \frac{dx}{dy} = \frac{1}{\sqrt{1-y^2}}$

B) $S = \int_0^1 2\pi y \sqrt{1 + (\frac{dx}{dy})^2}\, dy$

$\checkmark = 2\pi \int_0^1 y \sqrt{1 + (\frac{1}{\sqrt{1-y^2}})^2}\, dy$

$$S = 2\pi \int_0^1 y \sqrt{1 + \frac{1}{1+y^2}}\, dy$$

$\boxed{\#5}$

$$|E_M| = \frac{K(b-a)^3}{24n^2}$$

$K = f''(x)_{max}$

$f(x) = x^3 - x$

$f'(x) = 3x^2 - 1$

$f''(x) = 6x$

$f''(0) = 0$

$f''(1) = 6$ max

$$.0001 = \frac{\cancel{6}^{1}(1-0)^3}{\cancel{24}_{4} n^2}$$

$$n^2 = \frac{1}{4(.0001)} = \frac{1}{.0004} = \frac{1}{\frac{4}{10000}}$$

$$\sqrt{n^2} = \sqrt{\frac{10000}{4}}$$

4

$100 \times 100 = 10000$

$$n = \frac{100}{2} = 50$$

from the list $\{35, 40, 55, 70\}$ it would be 55.

$\boxed{55}$

#6

$$\lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right)\frac{\sqrt{3n^2+n}}{\sqrt{n^2-n+1}}$$

$$=\lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right)\frac{\sqrt{\frac{3n^2}{n^2}+\frac{n}{n^2}}}{\sqrt{\frac{n^2}{n^2}-\frac{n}{n^2}+\frac{1}{n^2}}}$$

$$=\lim_{n\to\infty} \cos\left(\frac{\pi}{n}\right)\frac{\sqrt{3+\frac{1}{n}}}{\sqrt{1-\frac{1}{n}+\frac{1}{n^2}}}$$

$$= \cos(0)\frac{\sqrt{3+0}}{\sqrt{1-0+0}}$$

3

$$=(1)\frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$$

$$\int_1^\infty \frac{\sin^2(x)\,dx}{\sqrt{x+x^7}}$$

$$0 \le \sin^2 x \le 1$$

$$\frac{\sin^2 x}{\sqrt{x+x^7}} \le \frac{1}{\sqrt{x+x^7}} \le \frac{1}{x^{7/2}}$$

Since $\int_1^\infty \frac{1}{x^{7/2}}$ converges by the p-test, $\int_1^\infty \frac{\sin^2 x}{\sqrt{x+x^7}}$ must also converge by the Comparison test